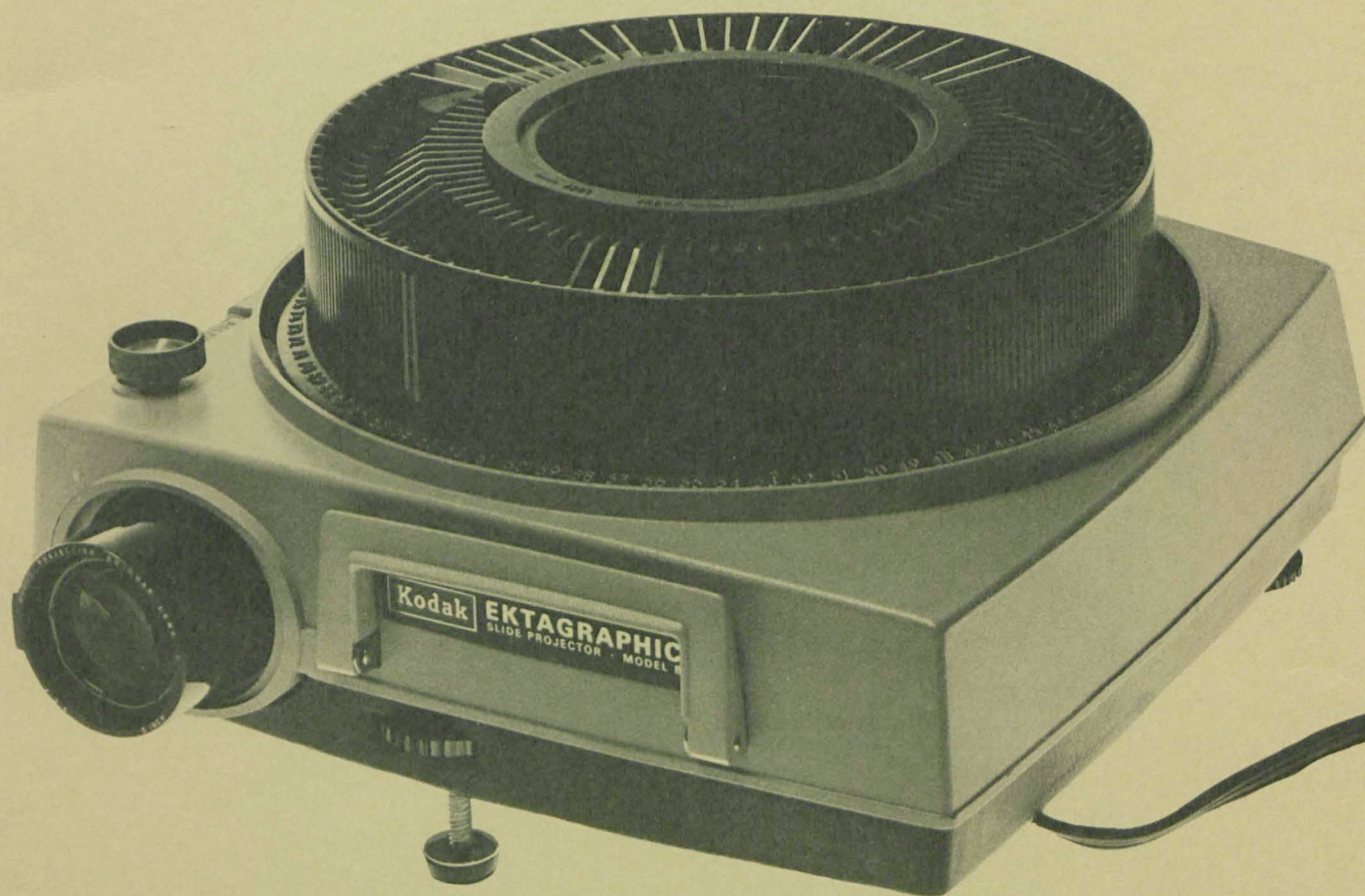


PHYSICS OF TECHNOLOGY

COORDINATED BY AMERICAN INSTITUTE OF PHYSICS



THE SLIDE PROJECTOR

Geometrical and Physical Optics

THE SLIDE PROJECTOR

A Module on Geometrical and Physical Optics

TERC

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Introduction

Section A

Section B

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The Slide Projector

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PREFACE

ABOUT THIS MODULE

Its Purpose

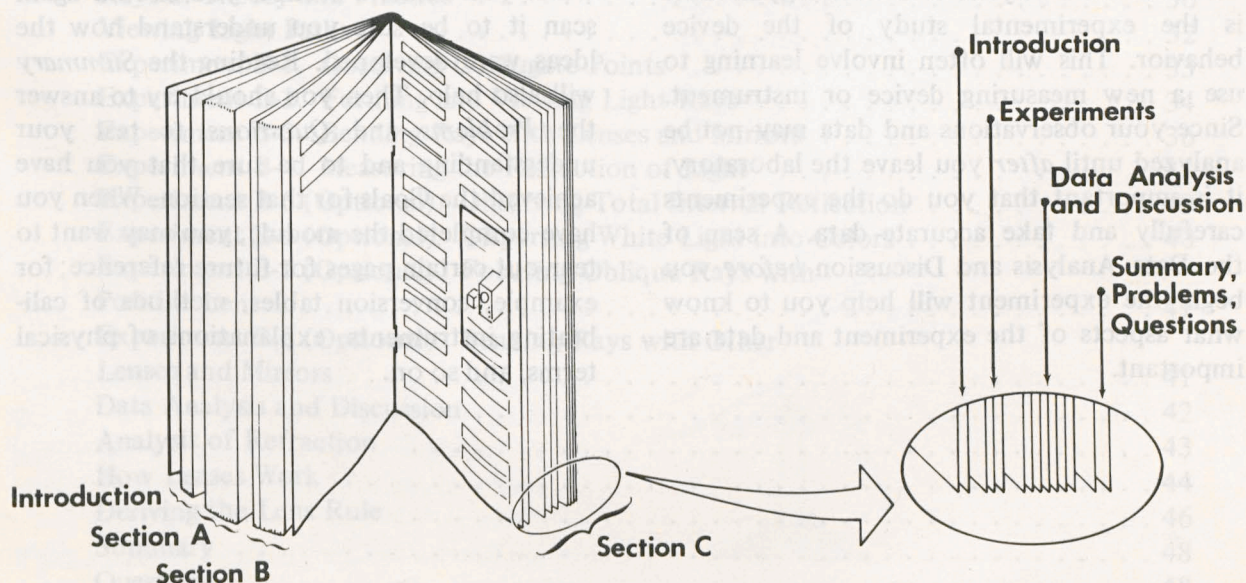
The purpose of the Physics of Technology program is to give you an insight into the physical principles that are the basis of technology. To do this you are asked to study various technological devices. These devices have been chosen because their operation depends on or illustrates some important physical phenomena. In this module the device is the slide projector. Its design and use involve many principles of optics.

The PoT program has adopted a modular format with each module focusing on a single device. Thus you can select only those modules that relate to your own interests or areas of specialty. This preface highlights some of the features of the modular approach so that you may use it efficiently and effectively.

Its Design

The *Introduction* explains why we have selected the slide projector to study and what physical principles will be illustrated in its behavior. Learning *Goals* are given, as well as *Prerequisite* skills and knowledge you should have before beginning. The three *Sections* of the module treat different aspects of the projector. They are of increasing difficulty, but each can be completed in about one week.

Each section begins with a brief *Introduction* to the topics treated and how they relate to the behavior of the device. The *Experiments* follow and take about two to three hours. The body of the section then describes the method of *Data Analysis* including a *Discussion* of the physical principles which explain your results. A *Summary* ends the section with *Problems* and *Questions*.



HOW TO USE THIS MODULE

To Begin

This module has been written so that it can be quickly and easily scanned. That is, you can get the gist of the ideas and experiments by simply flipping from page to page, reading only the headings and italicized words, and looking at the illustrations. We suggest that before you begin a section or an experiment, you scan through it in this way so that you will know where you are going.

EXPERIMENT A-4 (OPTIONAL): Working with Mirror Images

Spherical Mirror

When you examined the light bulb of the projector earlier, you found a small curved mirror located either behind the lamp or actually inside it (Figure 29).

David Wilson

The only basic difference is that, whereas the lens was placed in front of the object (pin stuck in a rubber stopper), now the mirror must be placed *beyond* it (Figure 28).



Figure 15. Braggster beam with transverse size



Figure 18. Move the object in front of the mirror.

A mirror of this shape is called a **spherical mirror**, because it is shaped like a section of a large sphere. Its purpose is to project a real image of the lamp filament near the actual filament, to make the light source brighter and more uniform.

A spherical mirror acts much like a projection lens. It can form real or virtual images, depending on how large the object distance is compared to the focal length. Thus, you can repeat Experiment A-2, using a mirror instead of the lens. You will need a fairly large mirror, with a focal length of several inches, which can be used to project a picture of a

Otherwise, the procedure is the same. Use the elimination of parallelism with respect to a reference line on a card behind the screen to locate the image. Then measure the distances from the pin and the line to the mirror.

EXPERIMENT A-5 (OPTIONAL): Projective 3-D Images with a Solenoid

As ordinary slide provides a visual record with one major defect. It shows only a single view of the object. Though you may try to alter the view by looking at the slide from different angles or positions, you are limited to the view originally seen by the camera.

Holography)

There is a method for making a visual record on a single, flat surface in which the viewpoint is flexible. This technique is called *holography*. The record, which is called a *hologram*, looks nothing like an ordinary slide, because there is no picture to be seen on it. Instead, there is a direct record of the original light in a special coded form, somewhat like the coded form of sound waves on a phonograph record. Attempting to see a picture by directly examining the hologram is

Wavefront Reconstruction

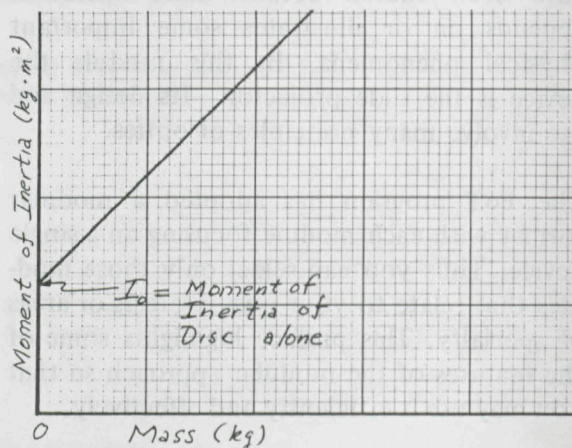
The process of using a hologram to recreate a visual image is called *nonfront reconstruction*. In effect, the light patterns which came from the original object are reproduced when light is passed through a hologram. Viewing



Figure 10: Values of contributions for cases of *Acetabulum*.

The Data Analysis

The data you take will generally have to be graphed before the results can be analyzed. Graphing and graphical analysis are essential parts of experimental science. Understanding graphs also is important for technology since technical information is often presented graphically. For these reasons, and since the discussion of your results will be centered around your graphs, it is important that you prepare them clearly and accurately.



The Experimental Activities

The heart of a Physics of Technology module is the experimental study of the device behavior. This will often involve learning to use a new measuring device or instrument. Since your observations and data may not be analyzed until *after* you leave the laboratory, it is important that you do the experiments carefully and take accurate data. A scan of the Data Analysis and Discussion *before* you begin the experiment will help you to know what aspects of the experiment and data are important.

Review

When you finish a section you should again scan it to be sure you understand how the ideas were developed. Reading the *Summary* will also help. Then you should try to answer the *Problems* and *Questions* to test your understanding and to be sure that you have achieved the Goals for that section. When you have completed the module, you may want to tear out certain pages for future reference; for example, conversion tables, methods of calibrating instruments, explanations of physical terms, and so on.

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The Slide Projector

Introduction: Why Study Projectors ?

Projectors Are Common In Daily Life

When you hear the word “projector,” you are apt to think of a device for making a picture of a slide or movie appear on a screen (Figure 1).

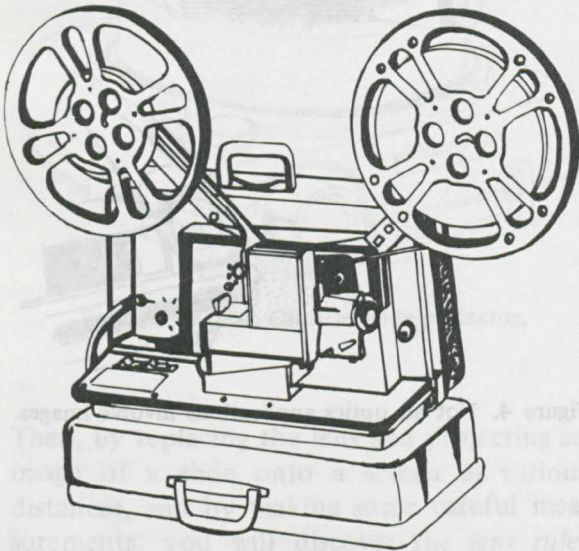


Figure 1. A movie projector.

However, did you know that a large variety of other common devices are, in a sense, also projectors, even though they are called by different names? For instance, a *camera* is really a projector turned inside out. The *camera lens* makes a picture of a scene outside the camera appear on a “screen” (the film) inside the camera itself. Such a picture of an object, formed by a lens, is called an *image*.

What about *binoculars*, *telescopes*, and *microscopes*? They are also basically projectors.

Their lenses project images of things too distant or too small to be easily viewed with the eye alone. Of course, the images produced by a telescope or microscope are not necessarily viewed on a screen. These images usually are viewed directly, using a lens (the *eyepiece*) as a magnifying glass. If desired, however, the images *can* be made to appear on a screen or film, as with other projectors.

Projectors Are Useful In Technology

Projectors are used in an enormous variety of industrial and scientific processes. For example, enlarging or reducing images by projection methods is essential in modern printing. Copying methods like xerography also start with a projector.

In another important application, high-quality projectors are used by the electronics industry to prepare *integrated circuits*. These circuits have made possible such developments as electronic computers which can fit into one’s shirt pocket or handbag.

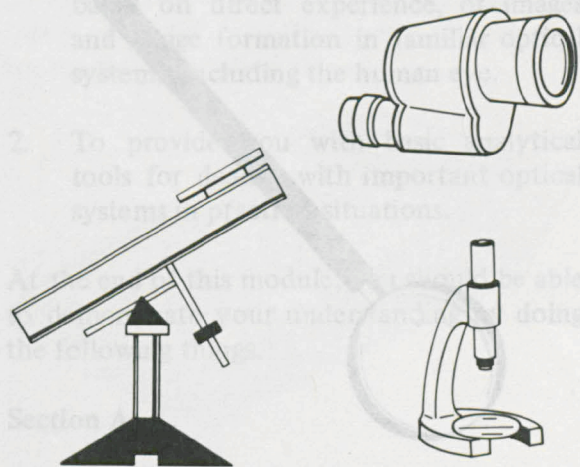


Figure 2. Other projectors.

Image-Forming Principles Are General

Although many image-forming devices are not projectors, they can be designed and understood using the same basic ideas of optics. Perhaps the most familiar and important example is that of ordinary *eyeglasses*.

Why are glasses ordinarily not “projectors”? This is because, as glasses, they form *virtual images*. Unlike a real image, a virtual image cannot be cast on a screen and then viewed by looking at the screen. It must be viewed by looking directly at the lens itself.

Another example is the inexpensive *slide viewer*. This is little more than a magnifier with a built-in light source. Since a magnifier also forms virtual images, the viewer is basically different from the projector.

Later in this module, you will learn how to distinguish between real and virtual images. You will find that the same simple ideas and methods of optics lead to an understanding of all the various devices which are in the category of “optical devices.”

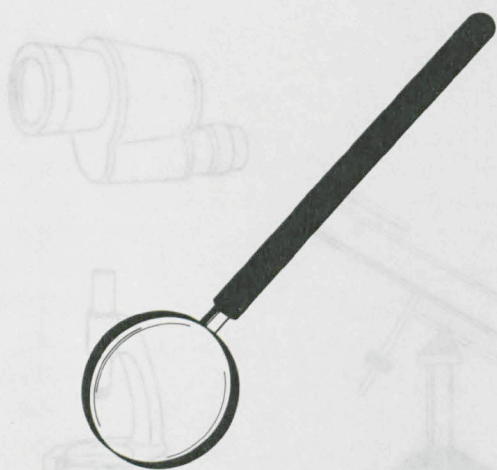


Figure 3. Not all optical devices are “projectors.”

Optical Science Has Wide Applications

In everyday life and in technology there are many applications of optics which do not involve images at all. Yet the same principles of physics apply to these applications.

Some gadgets, such as a flashlight, are so familiar that we may not regard them as optical devices. For instance, an automobile headlight is a fairly complicated optical device. It incorporates both a curved mirror and a lens to focus the light into a beam.

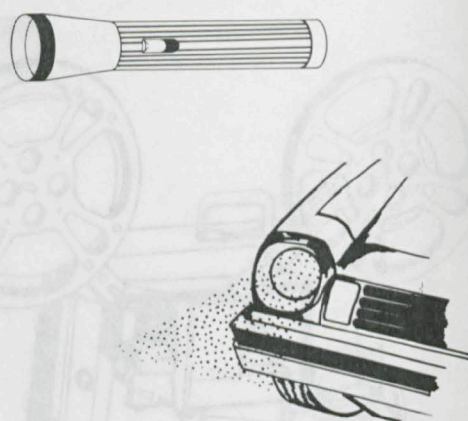


Figure 4. Not all optics applications involve images.

There are other examples of optical devices which are not so familiar but which are important in modern technology. For example, *fiber optics* devices are flexible “light pipes” which can be used to direct light around corners and into difficult places. You may have seen a decorative fiber optics lamp or even a computerized price reader at the supermarket which uses fiber optics. Another important technological device utilizing optics is the *spectrophotometer*. This helps in identifying materials from their characteristic emission or absorption of light. Spectrophotometers have wide uses in medicine, criminology, industrial chemistry, and many other fields.

WHAT WILL YOU LEARN?

SECTION A: Lenses and Images

Using an ordinary slide projector (Figure 5), you will first examine the elements of its basic optical system, including the *light source*, the *object*, and the *projecting lens*. By removing the lens and making a few observations, you will determine its *focal length* and the position of its *optical center*.

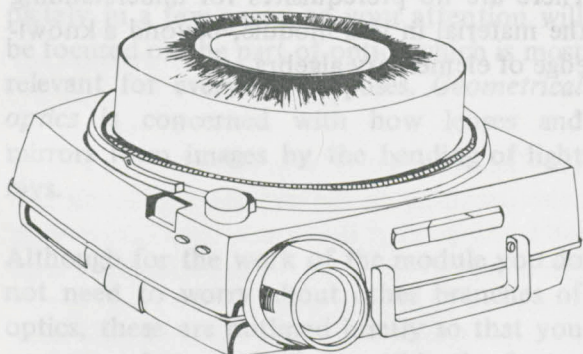


Figure 5. Kodak Carousel slide projector.

Then, by replacing the lens and projecting an image of a slide onto a screen at various distances, and by making some careful measurements, you will discover the *lens rule*. This relates the *image distance* and *object distance* to the *focal length* of the lens. The rule giving the *magnification* will also be determined.

Finally, by some further work with the projector's lens, as well as with other lenses and mirrors, the differences between *real images* and *virtual images* will be observed. This will be taken up further in Section B.

SECTION B: Light Rays and Their Behavior

Here you will examine in detail the process of image formation by the *reflection* and *refraction* of light. The paths of individual narrow beams of light ("rays") will be traced in

various cases involving lenses and mirrors. Smoke will be used to reveal the rays, and the meaning of the *focal length* and *focal plane* will be seen.

By carefully measuring how much bending of light occurs when a single ray meets a mirror, you will learn about the *law of reflection*. Similarly, by measuring the bending when the ray goes from air into another transparent medium, you will determine the *law of refraction*. You will also be able to determine the *index of refraction* of various substances, including glass and water.

A Note about Equipment

Throughout this module we will illustrate the Kodak *Carousel* projector (see Figure 5). However, almost any slide projector can be used for the experiments, and yours may differ from the one shown. The basic optical properties and components will be the same with only differences in design detail. You should use the first experiment to familiarize yourself with these differences.

GOALS

This module has two general goals:

1. To give you an improved understanding, based on direct experience, of images and image formation in familiar optical systems, including the human eye.
2. To provide you with basic analytical tools for dealing with important optical systems in practical situations.

At the end of this module, you should be able to demonstrate your understanding by doing the following things.

Section A

1. Determine the focal length of a thin lens by simple measurements.

2. Locate real images using a projection method.
3. Locate virtual images using a parallax method.
4. Calculate real and virtual image distances for a positive lens using the universal lens rule.
5. Calculate the magnification of images by knowing the object and image distances.
6. Predict the power of simple lens combinations, including positive and negative lenses.

Section B

1. Collimate a light beam using a slit source and projection lens.

2. Use ray tracing methods to investigate the laws of reflection and refraction.
3. Measure the index of refraction of water.
4. Calculate the critical angle for total internal reflection of light.
5. Derive the universal lens rule using arguments based on ray tracing.

PREREQUISITES

There are no prerequisites for understanding the material in this module, beyond a knowledge of elementary algebra.



SECTION A

Lenses and Images

INTRODUCTION

Geometrical Optics

In this module the projector is used to help you study optics. Optics is the study of light and all of the effects produced by light. That is obviously too much to be covered completely in a few pages, so your attention will be focused on the part of optics which is most relevant for everyday purposes. *Geometrical optics* is concerned with how lenses and mirrors form images by the bending of light rays.

Although for the work of the module you do not need to worry about other branches of optics, these are outlined briefly so that you may see what you will do within the framework of the "big picture."

Physical Optics

The non-geometrical branches of optics go into the underlying nature of light itself. *Physical optics* is the study of phenomena

which depend on the fact that light is a wave. For instance, a light beam which goes through a narrow opening "spreads" out on the other side. This spreading is called *diffraction*, and it is a property of all waves. (See Figure 6.)

Many practical devices depend on physical optics. Perhaps you have heard of the *hologram*, a device for making images appear without the use of any mirrors or lenses. Later you will have an opportunity to see the remarkable effects produced by a hologram, and to compare them with effects of geometrical optics.

Quantum Optics

Probing still deeper, *quantum optics* deals with the basic nature of light at the atomic level. The primary aim of quantum optics is to provide an understanding of how light is produced and how it interacts with matter on an atomic scale.

The invention of the laser was one of the practical successes of quantum optics. Besides the laser, many familiar devices also depend on quantum optical principles. (See Figure 7.) These include "electric eyes," fluorescent lamps, and TV picture tubes.

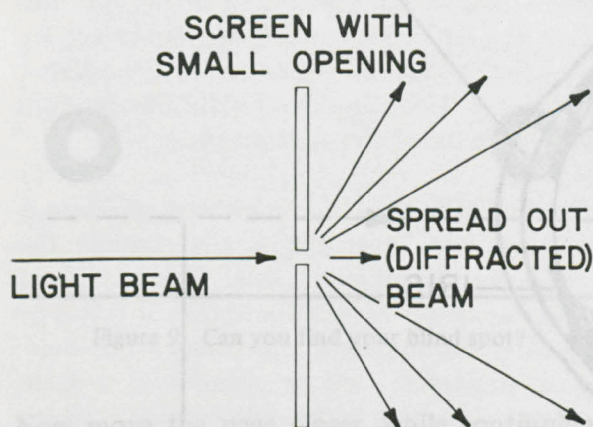


Figure 6. Physical optics deals with wave properties of light.



Figure 7. The numerical displays of electronic calculators use devices that depend on quantum optics.

HUMAN OPTICS

While humans have five senses (touch, taste, smell, hearing, and vision), the sense of vision seems most important. Perhaps our most direct contact with the world is through visual impact. The eyes are like “windows,” through which our self looks out at the world.

Most optical devices are designed to provide ways of overcoming limitations of human vision. Thus, it is worth spending a few moments to consider some of those limitations. First, it is necessary to consider the nature of the visual mechanism.

Lenses and Images in the Eye

You know, of course, that the eyes are not really like windows. In many ways, the eye is similar to a television camera.

Just as in the TV camera, a lens in the eye

projects an image onto a light-sensitive screen. In the eye this is the *retina*. (See Figure 8.) When light strikes the retina, electrical impulses are created. These carry information about the image, via the optic nerve, to the brain. The optic nerve is thus like the cable which carries electrical impulses from the TV camera to a videotape recorder.

Visual Field

Many of our visual limitations are so familiar that we hardly give them a thought. For instance, you cannot see in all directions at once. The total range of directions in which you can see is called the *visual field* or *field of view*.

The field of clear view is restricted to a small cone surrounding the direction in which you are looking. That is because only one spot on the retina, the *fovea* (see Figure 8), has a large density of light receptors.

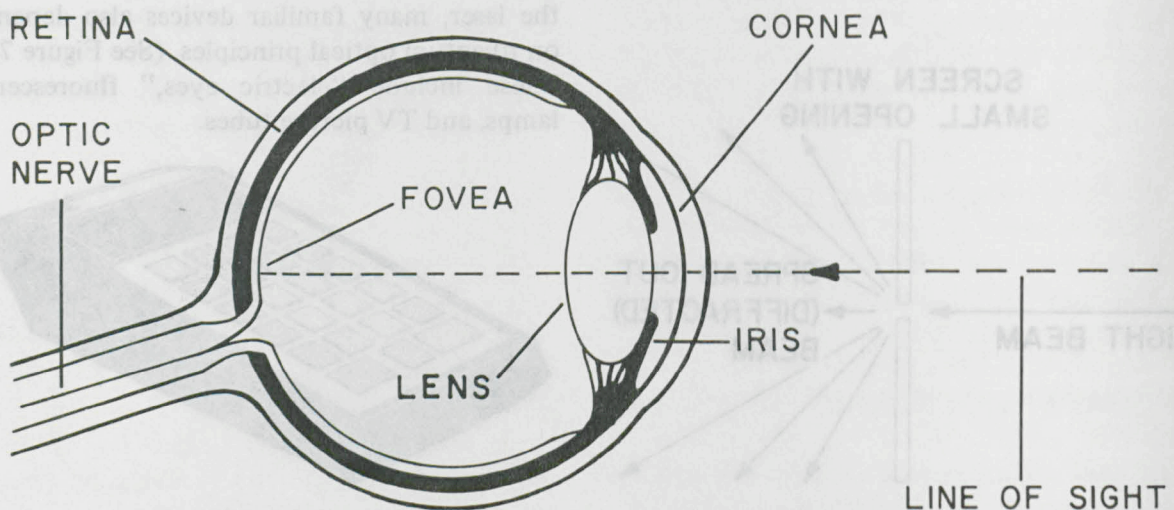


Figure 8. A cross section of the human eye. The lens forms an image on the fovea.

Sensitivity

The *sensitivity* of the eye is also limited. It is perhaps surprising that your eyes are most sensitive to light coming from the edge of the visual field, instead of from the center. Try looking for very dim stars at night. You can detect them much more easily when they are off to the side of the point at which you are looking directly. However, you can't see them very clearly. One purpose of telescopes and binoculars is to extend this limited sensitivity of the eyes. Large lenses or mirrors have greater *light-gathering power* than does the eye, because the light falls upon a larger useful area.

Blind Spot

Do you know that within the visual field of each eye there is a "hole" called the blind spot? Anything at the blind spot of one eye is completely invisible to that eye. It can be seen only because the blind spot of the other eye is in a different direction.

You can use the figure below to find the blind spot for each of your eyes. Close the left eye, for instance, and with the *right* eye look steadily at the *left* spot. You will see the other spot out of the corner of your right eye.

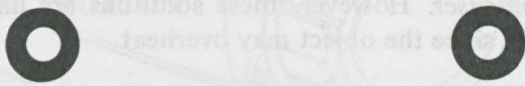


Figure 9. Can you find your blind spot?

Now move the page closer, while continuing to gaze at the left spot with the right eye. You should find that at a distance between 20

and 25 cm, the second spot suddenly disappears! You can see it by opening both eyes at once, or shifting your gaze. A clue to the reason for the blind spot was shown in Figure 8. Can you see it? (Hint: there is in effect a hole in the "screen" of the eye.)

Near Point

If you hold something in front of your face and bring it steadily closer, the image finally blurs. No amount of effort to see clearly will sharpen the image within a certain distance, called the *near point*. For most people, this is about 25 cm.

Try the above test, and have a friend check your near point with a ruler. To cope with this limitation—to examine an object more "closely" than is possible ordinarily—you use a magnifier. Later you will see how that works.

Visual Defects

So far, several visual limitations have been mentioned. Some can be overcome by artificial means; others cannot. However, these are all *basic* limitations in the sense that they are shared by all people. Visual limitations which are not basic are called *visual defects*.

For example, the near point may be abnormally distant—as much as one meter or more away. There is a natural tendency for people to develop this condition, called *farsightedness*, starting at about age forty.

The opposite difficulty is called *nearsightedness*. Here there is a "far point" beyond which nothing can be seen clearly. People who must use glasses for driving, but not for reading, have this condition.

Ordinarily there is *no* "far point" which limits vision, although there is always a near point. Why? Your work with lenses and images will give you an answer.

PROJECTOR SYSTEMS

A projector is designed to collect light from an object and re-form that light into a *real image* of the object. A real image is one which can be projected onto a screen.

There are three necessary elements of a projector system. These are:

1. Light source
2. Object
3. Appropriate lens or mirror

A simple projector is shown in Figure 10. That figure also identifies two important distances called the *object distance* and the *image distance*. As indicated, these distances are measured directly from the lens.

Opaque Projectors

In Figure 10, the light from the source does not go through the object, but is reflected from it. A projector which operates by reflected light is called an *opaque projector*. This is because the object is usually *opaque* (no light shines through it). Opaque projectors are convenient because almost any object can be “projected” if it is small enough to fit into the projector.

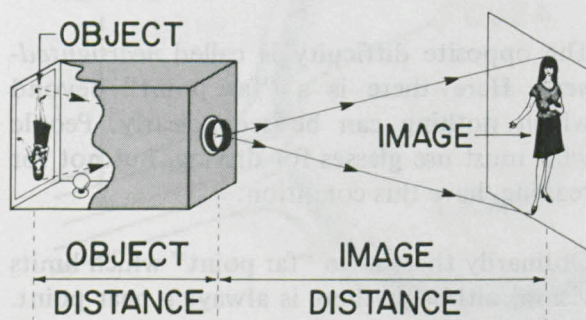


Figure 10. A simple projector.

Another type of “opaque projector” is sketched in Figure 11. This is an ordinary camera. The optical system of a camera is basically the same as that of a projector, the main difference being that the object and image positions are reversed. The light source is external (the sun in this case). Often, an artificial light source is used with a camera, as when shooting indoors using a flash attachment.

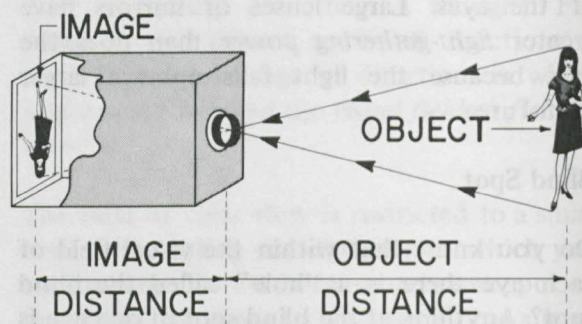


Figure 11. The camera is an opaque projector.

Transparency Projectors

A problem arises with opaque projectors in trying to form a bright enough image. Usually only a small fraction of the light from a source is reflected by an object. An extremely bright source can be used, and more of the light can be made to fall on the object by means of a secondary lens or mirror, called a *condenser*. However, these solutions are limited, since the object may overheat.

Another answer to the brightness problem is to pass light from the source *through* the object to the projector lens. This requires a transparent object. Thus a device of this type is called a *transparency projector*. The overhead projector used in schools is a good example. Another is the slide projector, which you will study in this module.

EXAMINING YOUR PROJECTOR

For this module you are provided with an ordinary slide projector of the type found commonly in the home. You may already have some idea of how slide projectors operate. In any case, you should take a few minutes to familiarize yourself with the projector you will be using.

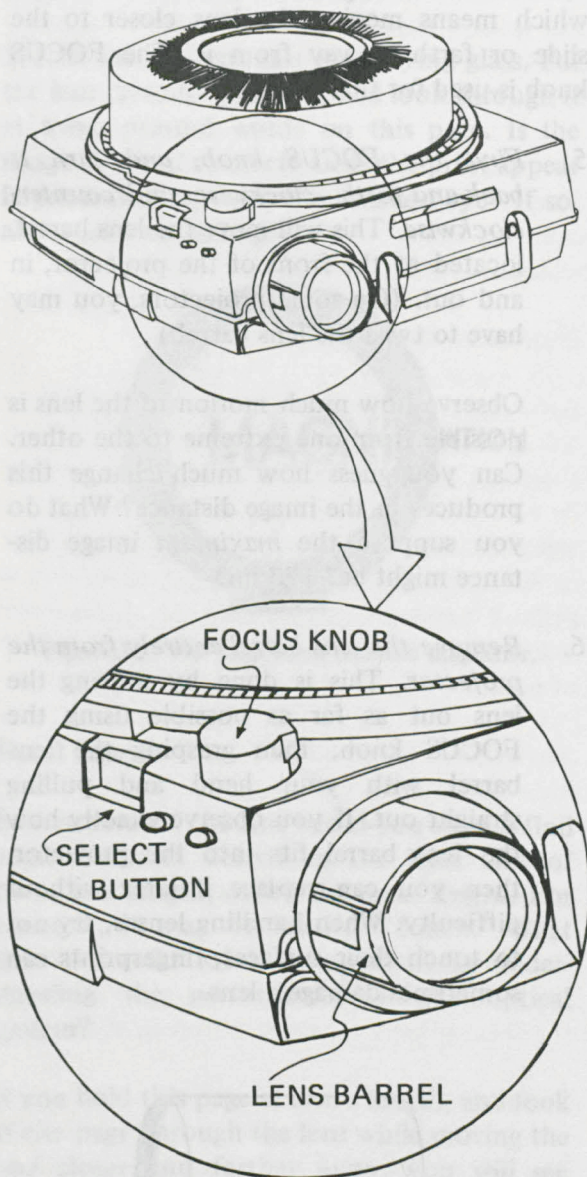


Figure 12. Closeup of some important features of the slide projector.

The Light Source

The incandescent lamp used as the light source is located near the rear of the projector. There is always some means of access for replacing the bulb if it burns out.

1. *Unplug the projector, and locate the bulb.* Notice that the bulb has several filaments. Their purpose is to enlarge the effective area of the source. How many filaments are there?

Notice also that there are some lenses between the bulb and the location of the slide. These are *not* projecting lenses.

Their purpose is to concentrate or to *condense* some of the light from the bulb onto the slide to increase the brightness of the final image. In most projectors, you also will see a curved mirror behind the lamp. It also is there to increase the amount of light reaching the slide. The lamp—condenser lens—mirror system is designed to spread the light as uniformly as possible across the entire area of the slide.

2. *Close the projector. Plug it in and switch it on.* In the better projectors, the switch is arranged so that whenever the light is on, the fan is also on. However, you can switch off the light and leave the fan running. Generally, the fan should be turned off at the same time as the light bulb, so that the optical components will cool slowly. The filament, however, is very fragile when it is hot, so the projector should never be moved when it is operating. If it has to be moved or the lamp must be changed immediately after operation, the fan may be used to cool it quickly after the bulb has been turned off.

The Object

The most common type of slide used in a projector of this type is a 2 in X 2 in slide, usually made from 35 mm film. The "35 mm" refers to the width of the film, as illustrated in Figure 13. The width of the cardboard mount is 2 in.

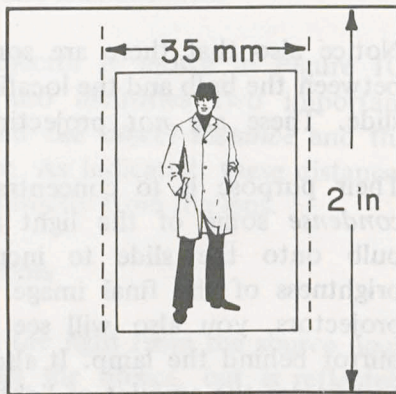


Figure 13. A typical 35 mm slide.

For the Carousel projector, many slides are held at once in a special container called the "carousel." The carousel fits over a spindle on top of the projector. When the appropriate control button is pushed to change slides, the old slide is automatically replaced in the carousel and a new one is inserted in the projector. You will not need the carousel here. You can insert a single slide and remove it by hand.

3. *Insert a slide into the projector.* Make sure the slide goes all the way in. However, do not force it. If it does not go in easily, you may not have placed it in quite the right position.
4. *Retrieve the slide.* On the Carousel this can be done by pushing the SELECT button. It is not necessary to reach down into the slot with your fingers to retrieve the slide.

The Projecting Lens

The lens of the slide projector is movable to allow a sharp image of the slide to be formed on a screen when the screen is at various distances from the projector.

Changing the distance to the screen changes the *image distance* of the projector. (See Figure 10.) The image can be sharply focused by deliberately changing the *object distance*, which means moving the lens closer to the slide or farther away from it. The FOCUS knob is used for that purpose.

5. *Find the FOCUS knob, and turn it back and forth, clockwise and counter-clockwise.* This will move the lens barrel, located at the front of the projector, in and out. (On some projectors, you may have to twist the lens barrel.)

Observe how much motion of the lens is possible from one extreme to the other. Can you guess how much change this produces in the image distance? What do you suppose the *maximum* image distance might be? 193 mi?

6. *Remove the lens barrel entirely from the projector.* This is done by moving the lens out as far as possible using the FOCUS knob, then grasping the lens barrel with your hand and pulling straight out. If you observe exactly how the lens barrel fits into the projector, then you can replace it later without difficulty. When handling lenses, try not to touch their surfaces; fingerprints can sometimes damage a lens.

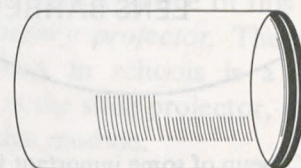


Figure 14. The projecting lens.

EXAMINING THE LENS

If you examine the projecting lens carefully, you will notice some things which require explanation. First, hold the lens up in front of you, at about arm's length, and look through it at some distant object using one eye. Do you see a clear image? Reverse the lens and try the same thing. Does this make any difference? Is the image *inverted* (upside down) or *erect* (right side up)?

Try to use the lens as a magnifying glass. Put the lens close to your eye, and look through it at some printed words on this page. Is the image inverted or erect? Does the print appear larger now than with the unaided eye? If so, about how much larger?



Figure 15. The projection lens as a magnifier.

Focal Length

Somewhere on the lens barrel you should find some numbers describing the lens. One of these is a length, called the *focal length*. For example, it may be labeled "5 inch." What does this refer to? Is it important for understanding the performance of the optical system?

If you hold this page at arm's length, and look at the page through the lens while moving the lens closer and farther away, you will see something interesting. The image will "flip over" at a certain point. Estimate how far the page is from the lens at the point this flipover occurs.

f-Number

Another specification you should find on the lens is the *f-number*. It appears as a ratio, in which the focal length f is divided by a particular number, such as 3.5. It may be written as either $f/3.5$ or $f:3.5$.

The *f-number* actually specifies the diameter of the lens opening, called the *aperture*. For instance, if the focal length $f = 5$ in and the *f-number* is $f/3.5$, the aperture is about 1.4 in. This is because $5 \text{ in} / 3.5 \approx 1.4 \text{ in}$.

The reason the *f-number* is important is that the brightness of an image depends on the aperture of the lens. For some cameras, in situations where the amount of available light and the type of film vary, the aperture is adjustable, and must be carefully chosen in order to deliver the proper amount of light to the film. However, the aperture of your projector's lens is fixed.

Lens Combination

Your projection lens actually contains several pieces of glass mounted together as a *lens combination*. (See Figure 16.) The most common combination used in projectors is the "Cooke triplet." It is made up of three individual lenses. For many purposes, the combination can be treated as one lens. You will see this in the first experiments.

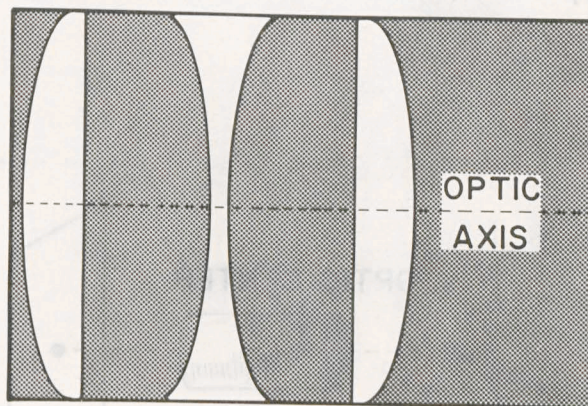


Figure 16. A good-quality lens barrel may contain a "triplet" of lenses.

EXPERIMENT A-1. Finding the Focal Plane

Background

When a lens forms an image of a very distant object, the image is located in a plane called the *focal plane*. The major task here is to find the focal planes of your projection lens.

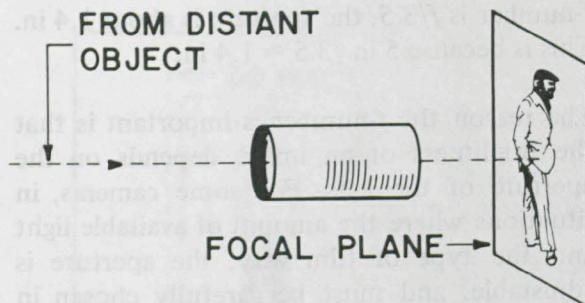


Figure 17. The focal plane is the place where a distant object is imaged.

There are really two focal planes, one on each side of the lens, as indicated in Figure 18. This is because the lens can form images using light coming from either side. Note that the lens barrel is *not* centered between the focal planes.

A point midway between the two focal planes, on a line along the axis of the lens (*optic axis*), is called the *optic center*.

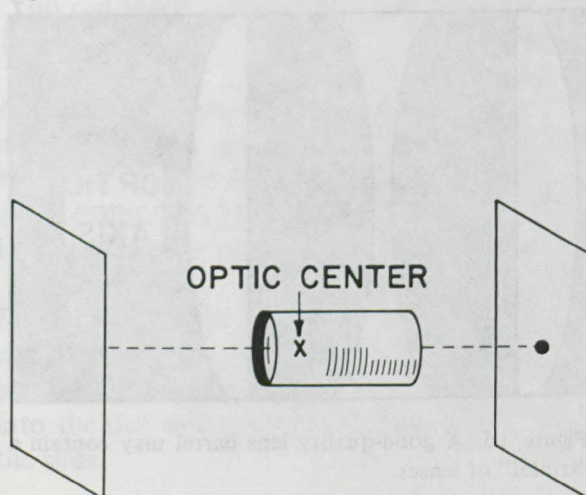


Figure 18. There are two focal planes.

By the way, the focal length is (approximately) the distance from the optic center to either focal plane. Thus, if the two focal planes are separated by a total distance D , the focal length can be calculated as $f = D/2$. This approximation is true for a single "thin" lens and for lens combinations, such as your projection lens, in which the separation between lenses is sufficiently small. Thick lenses and widely separated combinations require a more careful treatment, and will not be dealt with here.

This experiment will permit you to find the focal length of your lens by using the above equation. You can also find the optic center, which will be needed in later experiments.

Projection Method

Basically, the experiment is very simple. You will mount the lens near a screen on which the sharp image of a very distant object can be viewed. The term "very distant" means far away compared to the focal length f . A light in the ceiling is a convenient object, and a distant scene outside would be even more exact. The screen can be a white card or slip of paper.

At first, the image of the light will be "out of focus." That is, it will appear fuzzy because the screen is not at the proper distance from the lens. Only when the screen is at the proper distance will the image be sharp.

The focal plane moves with the lens. When the image of a distant object appears sharpest, the focal plane is at the position of the screen.

Setup

The setup for the experiment is shown in Figure 19. The lens barrel is held above the lab table with a clamp.

CAUTION: Make sure the lens is held securely in the clamp. If it falls, it may easily break.

The lens barrel should be aimed at a brightly lighted object that is far away, at least 15 ft. The object can be something outside a window. Or, you can turn on the projector, without the lens, and use it to illuminate an object. In either case, the farther the object is from the lens, the more accurate will be your result.

Procedure

1. *Locate the image on the screen.* If you don't see an image, either the lens is much too close to the screen or much too far away.
2. *Focus the image sharply* by moving the screen back and forth. The image will be in focus only over a narrow range of distances.
3. *Measure and record the distance from the front of the barrel to the screen.* This is not the focal length, of course, since it is not measured from the optic center.
4. *Turn the lens over, end to end, and repeat steps 1-3.* This locates the other focal plane. The distance must be measured from the *same end* of the barrel as for step 3.

Final Step

Later, you will need to know the optic center of your lens, since object and image distances can be simply related if they are measured from the optic center. Also, the focal length of the lens is defined as the distance from the optic center to the focal planes.

5. *Determine the optic center of the lens and mark it on the barrel.* Use your measured distances to each focal plane to make your determination. The location of the optic center can be marked by putting a piece of tape on the lens barrel and marking it with a pencil.

To make sure you have found the optic center, *repeat steps 1-4 above*, this time measuring from the optic center. The two focal plane distances should now be equal.

6. *Record in the data table the distance from the optic center to each of the focal planes* as the focal length of the projector lens. If the focal length is marked on the lens barrel, compare the two values. Do they agree?

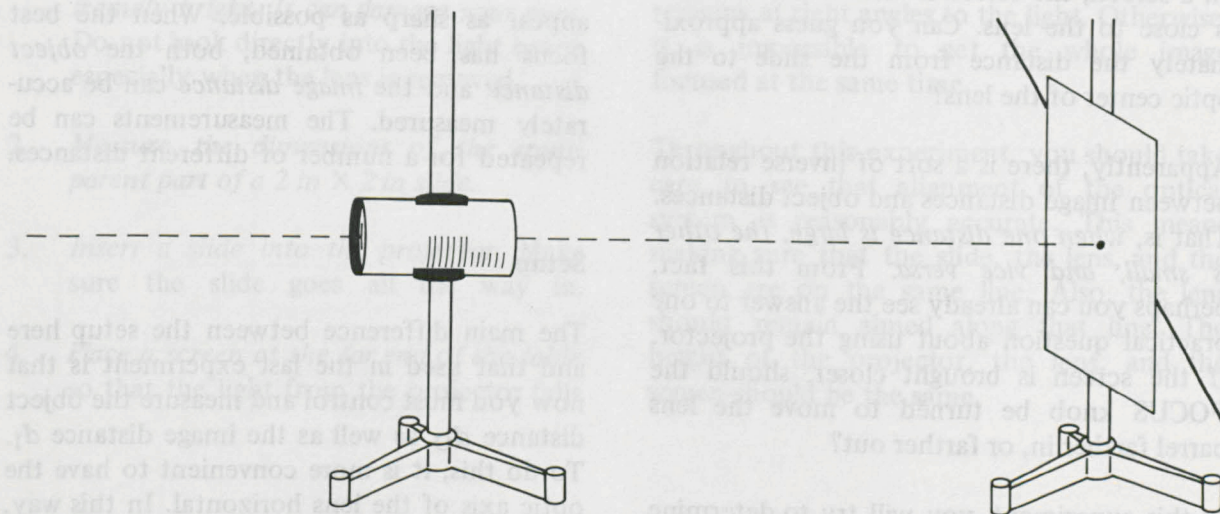
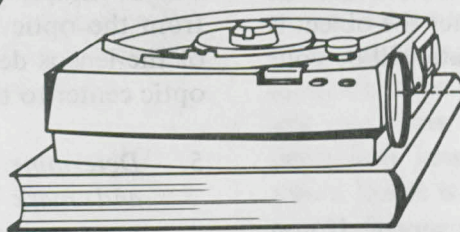


Figure 19. Form an image of a distant object to determine the focal plane position.



EXPERIMENT A-2. Determining Image Distances

Aim of the Experiment

In Experiment A-1, you used a lens to form an image of a distant object. This image was located quite close to the focal plane of the lens.

Ordinarily, the situation is just the reverse. When a slide projector is used to show slides on a screen, the screen is distant and the slide is close to the lens. Can you guess approximately the distance from the slide to the optic center of the lens?

Apparently, there is a sort of inverse relation between image distances and object distances. That is, *when one distance is large, the other is small, and vice versa*. From this fact, perhaps you can already see the answer to one practical question about using the projector. If the screen is brought closer, should the FOCUS knob be turned to move the lens barrel farther in, or farther out?

In this experiment, you will try to determine more precisely the relation between image distances and object distances for your lens.

Method

The method to be used is similar to that used in Experiment A-1. You will set up the lens on a stand, at a certain distance from an illuminated object, and project the image formed by the lens on a nearby screen.

Having found the image, you will move the screen back and forth to make the image appear as sharp as possible. When the best focus has been obtained, both the *object distance* and the *image distance* can be accurately measured. The measurements can be repeated for a number of different distances.

Setup

The main difference between the setup here and that used in the last experiment is that now you must control and measure the object distance d_O as well as the image distance d_I . To do this, it is more convenient to have the optic axis of the lens horizontal. In this way, the object can also be located on the lab table.

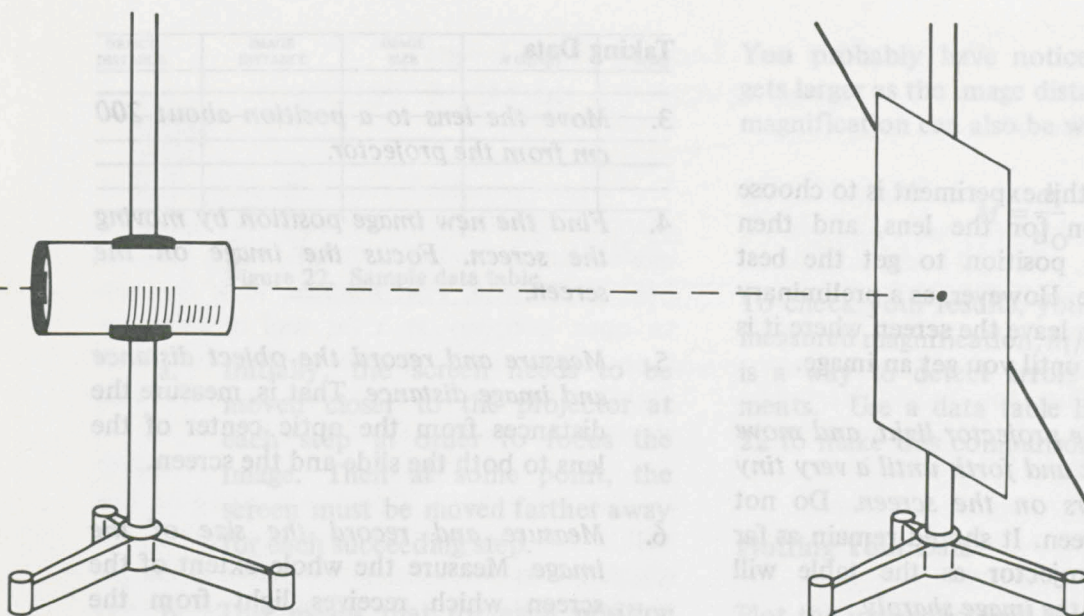


Figure 20. Setup for projecting images.

The light source used will be the projector light, and the object will be an ordinary slide.

1. *Place the projector at one end of the lab table.* Aim it so that if the lens were in place, the image would be projected beyond the far end of the table. The aim should be as nearly level as possible.

CAUTION: *The projector light is extremely bright. It can damage your eyes. Do not look directly into the light beam, especially when the lens is removed.*

2. *Measure the dimensions of the transparent part of a 2 in X 2 in slide.*
3. *Insert a slide into the projector.* Make sure the slide goes all the way in.
4. *Place a screen at the far end of the table* so that the light from the projector falls

squarely on it. The screen should be six or eight feet from the projector and perpendicular to the light beam.

5. *Set up the lens on the ring stand directly in front of the projector.* The optic axis of the lens should be level and pointed straight from the projector to the screen.

As you proceed, make certain that the screen remains at right angles to the light. Otherwise, it is impossible to get the whole image focused at the same time.

Throughout this experiment, you should take care to see that alignment of the optical system is reasonably accurate. This means making sure that the slide, the lens, and the screen are on the same line. Also, the lens should remain aimed along that line. The height of the projector, the lens, and the screen should be the same.

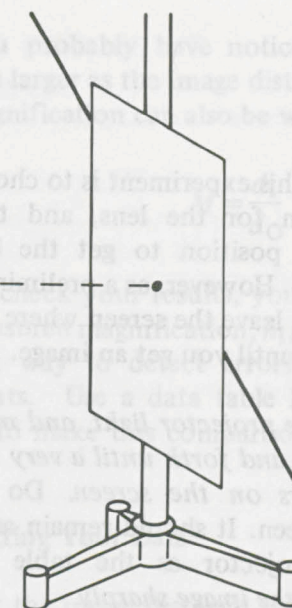
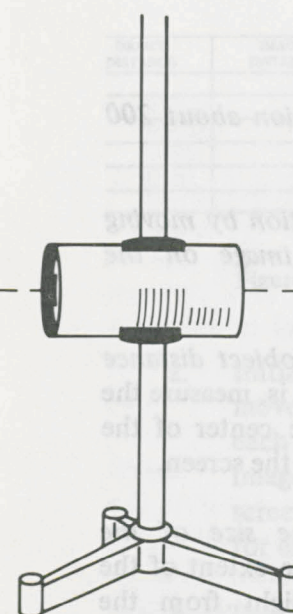
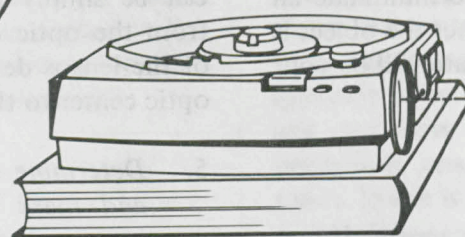


Figure 20. Setup for projecting images.

EXPERIMENT A-2. Determining Image Distances

Aim of the Experiment

In Experiment A-1, you used a lens to form an image of a distant object. This image was located quite close to the focal plane of the lens.

Ordinarily, the situation is just the reverse. When a slide projector is used to show slides on a screen, the screen is distant and the slide is close to the lens. Can you guess approximately the distance from the slide to the optic center of the lens?

Apparently, there is a sort of inverse relation between image distances and object distances. That is, *when one distance is large, the other is small, and vice versa*. From this fact, perhaps you can already see the answer to one practical question about using the projector. If the screen is brought closer, should the FOCUS knob be turned to move the lens barrel farther in, or farther out?

In this experiment, you will try to determine more precisely the relation between image distances and object distances for your lens.

Method

The method to be used is similar to that used in Experiment A-1. You will set up the lens on a stand, at a certain distance from an illuminated object, and project the image formed by the lens on a nearby screen.

Having found the image, you will move the screen back and forth to make the image appear as sharp as possible. When the best focus has been obtained, both the *object distance* and the *image distance* can be accurately measured. The measurements can be repeated for a number of different distances.

Setup

The main difference between the setup here and that used in the last experiment is that now you must control and measure the object distance d_O as well as the image distance d_I . To do this, it is more convenient to have the optic axis of the lens horizontal. In this way, the object can also be located on the lab table.

The light source used will be the projector light, and the object will be an ordinary slide.

1. *Place the projector at one end of the lab table.* Aim it so that if the lens were in place, the image would be projected beyond the far end of the table. The aim should be as nearly level as possible.

CAUTION: *The projector light is extremely bright. It can damage your eyes. Do not look directly into the light beam, especially when the lens is removed.*

2. *Measure the dimensions of the transparent part of a 2 in X 2 in slide.*
3. *Insert a slide into the projector.* Make sure the slide goes all the way in.
4. *Place a screen at the far end of the table* so that the light from the projector falls

squarely on it. The screen should be six or eight feet from the projector and perpendicular to the light beam.

5. *Set up the lens on the ring stand directly in front of the projector.* The optic axis of the lens should be level and pointed straight from the projector to the screen.

As you proceed, make certain that the screen remains at right angles to the light. Otherwise, it is impossible to get the whole image focused at the same time.

Throughout this experiment, you should take care to see that alignment of the optical system is reasonably accurate. This means making sure that the slide, the lens, and the screen are on the same line. Also, the lens should remain aimed along that line. The height of the projector, the lens, and the screen should be the same.

PROCEDURE

Finding the Image

The procedure in this experiment is to choose a definite position for the lens, and then adjust the screen position to get the best focus of the image. However, as a preliminary to the data taking, leave the screen where it is and move the lens until you get an image.

1. *Switch on the projector light, and move the lens back and forth until a very tiny image appears on the screen. Do not move the screen. It should remain as far from the projector as the table will allow. Focus the image sharply.*
2. *Measure the distance from the lens to the screen. Be sure to measure from the optic center of the lens which you marked.*

Does the distance measured here agree with the focal length measured in Experiment A-1?

How do you account for the difference?

Is the image upright or inverted?

Why do you think it is so small?

Taking Data

3. *Move the lens to a position about 200 cm from the projector.*
4. *Find the new image position by moving the screen. Focus the image on the screen.*
5. *Measure and record the object distance and image distance. That is, measure the distances from the optic center of the lens to both the slide and the screen.*
6. *Measure and record the size of the image. Measure the whole extent of the screen which receives light from the slide. Choose either the horizontal or vertical dimension.*
7. *Repeat steps 3-6 for decreasing distances to the projector in steps of about 10 to 20 cm. You should continue to make measurements until the lens is so close to the slide that the image is formed beyond the edge of the lab table. For the last data point you will have to reinsert the barrel in the projector.*

As you move the lens closer to the projector, you should notice some interesting features:

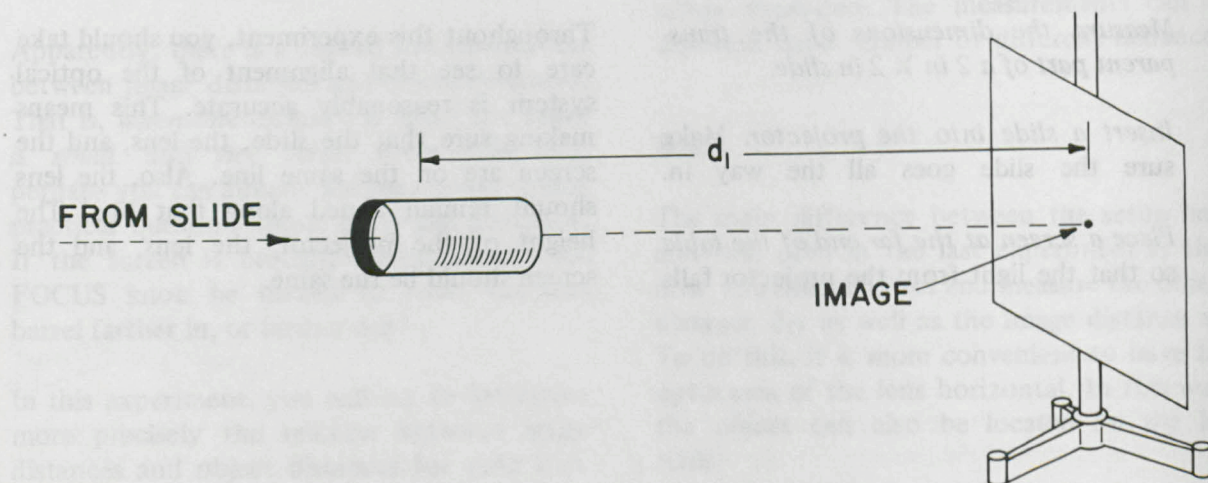


Figure 21. Measuring the image distance.

OBJECT DISTANCE	IMAGE DISTANCE	IMAGE SIZE	$M (h_I/h_O)$	d_I/d_O

Figure 22. Sample data table.

- a. Initially, the screen needs to be moved closer to the projector at each step in order to focus the image. Then at some point, the screen must be moved farther away for each succeeding step.
 - b. This means that for each position of the screen, there are *two* lens positions which will produce a sharp image.
 - c. Also, there is a minimum position of the screen—the one closest to the projector—for which a sharp image can be produced. If the screen is closer to the slide than this, no real image can be formed by the lens.
8. *Find the two lens positions which produce a sharp image* for some convenient screen position, say 100 cm, from the slide. Measure and record the distances and the image sizes.
 9. *Find the minimum screen position for which only one image can be produced.* Measure and record the distances and the image size.

Calculating the Magnification

One way to check your results makes use of the *magnification*, M . This is defined as the ratio of image size to object size:

$$M = \frac{h_I}{h_O}$$

where h_I is the height of the image and h_O that of the object. The magnification may be greater than 1 (large image) or less than 1 (small image).

You probably have noticed that the image gets larger as the image distance increases. The magnification can also be written as:

$$M = \frac{d_I}{d_O}$$

To check your results, you may compare the measured magnification, h_I/h_O , to d_I/d_O . This is a way to detect errors in your measurements. Use a data table like that of Figure 22 to make this comparison.

Plotting Your Data

Plot the image distance versus object distance on a piece of ordinary graph paper. This will show the relation between d_O and d_I .

1. *Draw and label the two axes*, as in Figure 23. Select suitable scales for the axes so that the plot will fit on the paper and yet not be too small.
2. *Locate and mark each data point* with a dot. The dots may be enclosed in small circles for better visibility.
3. *Draw a smooth curve through the points.* Check any points which are very far off the line to see whether they are misplotted.

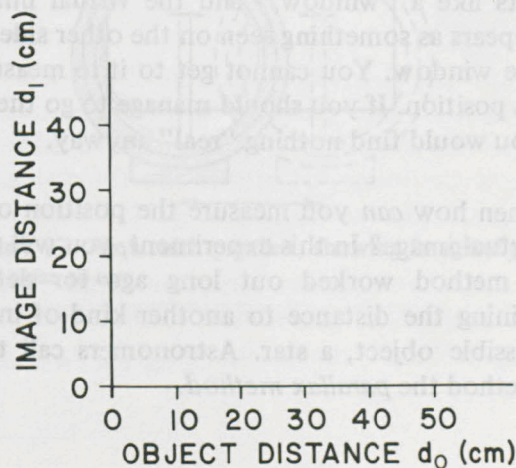


Figure 23. Sample graph.

EXPERIMENT A-3. Locating Virtual Images

Where Is a Virtual Image?

It is easy to determine the location of a real image because a real image can be focused on a screen. However, a *virtual* image cannot be cast onto a screen at all. This kind of image is formed when a lens is used as a simple magnifier. The object is placed very close to the lens, closer than the focal plane. The image can be seen only by looking directly through the lens itself. (See Figure 24.)

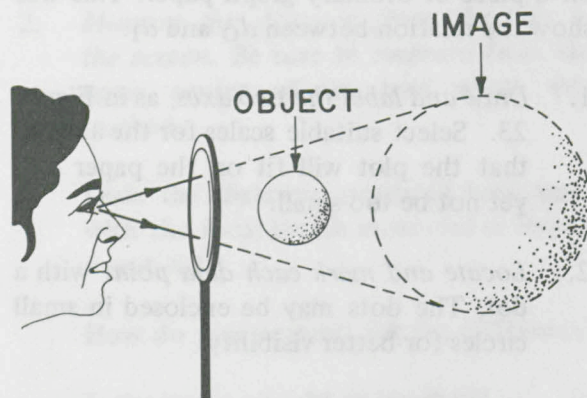


Figure 24. Viewing a virtual image.

A similar effect occurs when you see an image in a mirror. In such cases, the mirror (or lens) acts like a “window,” and the virtual image appears as something seen on the other side of the window. You cannot get to it to measure its position. If you should manage to go there, you would find nothing “real” anyway.

Then how *can* you measure the position of a virtual image? In this experiment, you will use a method worked out long ago for determining the distance to another kind of inaccessible object, a star. Astronomers call this method the *parallax method*.

Parallax Method

The term “parallax” refers to the apparent change in location of something seen against a more distant background when the line of sight is changed. For instance, look through an open doorway at a far wall of the next room. Then move your head back and forth slightly. The doorway appears to move with respect to the wall. (See Figure 25.)

On the other hand, a picture hanging on the wall itself does not appear to move. This suggests that the *absence* of parallax can be used to indicate that two things are at the same distance from the observer.

In the present application, you will look at a virtual image formed by your projector's lens. At the same time, you will view some real object placed behind the lens on the same line as the image. By adjusting the position of the reference object until all parallax is eliminated, you will be able to establish the image position.

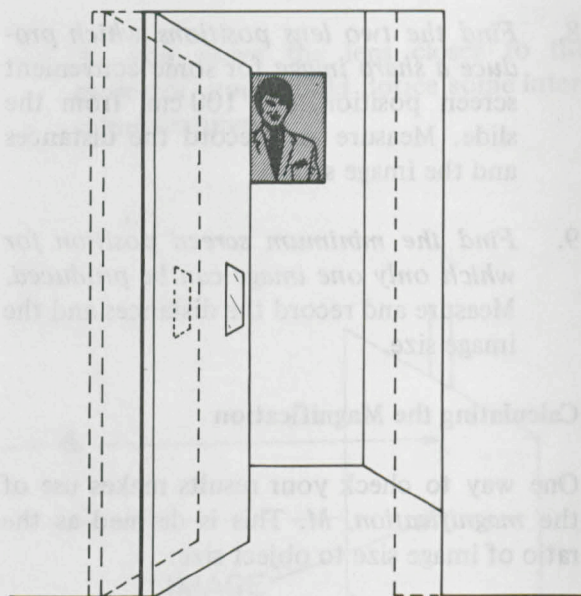


Figure 25. Because of parallax, when you move your eye, the doorway appears to move relative to a far wall.

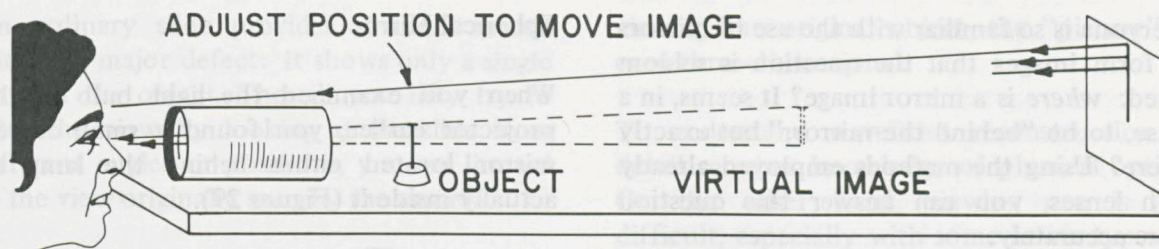


Figure 26. Move the object until the image and the line on the card are at the same position.

Setup

The projection lens should be placed on the lab table near one edge so that you can easily look into it (Figure 26). A convenient object for this purpose is a straight pin stuck into a rubber stopper. When the stopper is placed on the table, the pin should be vertical.

The image of the pin will be viewed at the same time as a vertical line drawn on a card. As shown in the figure, the card should be located well beyond the position of the pin. The card can be taped to a ring stand or to a book for support.

Procedure

Start with the pin close to the lens and the card about one meter away.

1. *Look at the pin through the lens with one eye while, with the same eye, looking just over the lens at the card.* It is not necessary to close the other eye, but it does no harm. The view should appear more or less as in Figure 27.
2. *Observe parallax by moving your eye from side to side.* Watch both the reference line and the image of the pin.
3. *Eliminate parallax by adjusting the*

image distance. Moving the pin also moves its image. At the correct image distance, there will be no apparent motion between the image and the reference line as you move your head from side to side.

Measure and record the object and image distances. Remember to make all measurements from the optic center.

4. *Repeat steps 1-4 for two or three other distance combinations.* In each case move the card closer, then follow the steps. In at least one case, have the card quite close, say 10 cm.

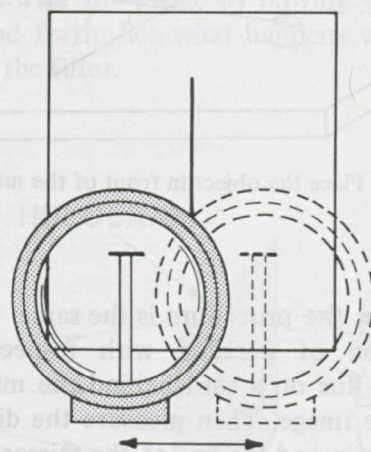


Figure 27. View with parallax, showing image of pin too close to eye.

EXPERIMENT A-4 (OPTIONAL). Working with Mirror Images

Everyone is so familiar with the use of mirrors to form images that the question is seldom asked: *where* is a mirror image? It seems, in a sense, to be “behind the mirror,” but exactly where? Using the methods employed already with lenses, you can answer this question more accurately.

Plane Mirror

An ordinary mirror is called a *plane mirror* because the reflecting surface is flat. A plane mirror forms virtual images. You can repeat Experiment A-3, substituting a plane mirror for the lens, to find the image position.

The only basic difference is that, whereas the lens was placed in front of the object (pin stuck in a rubber stopper), now the mirror must be placed beyond it (Figure 28).

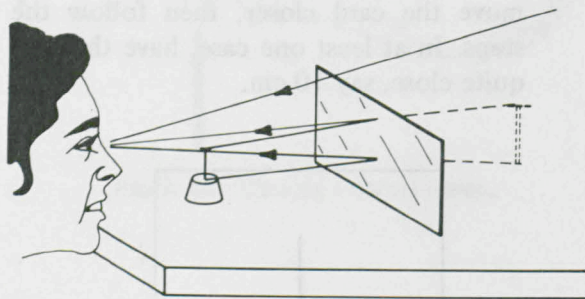


Figure 28. Place the object in front of the mirror.

Otherwise, the procedure is the same. Use the elimination of parallax with respect to a reference line on a card behind the mirror to locate the image. Then measure the distances from the pin and the line to the mirror.

The relation between the object distance and the image distance in this case is especially simple. What is that relation?

Spherical Mirror

When you examined the light bulb of the projector earlier, you found a small curved mirror located either behind the lamp or actually inside it (Figure 29).

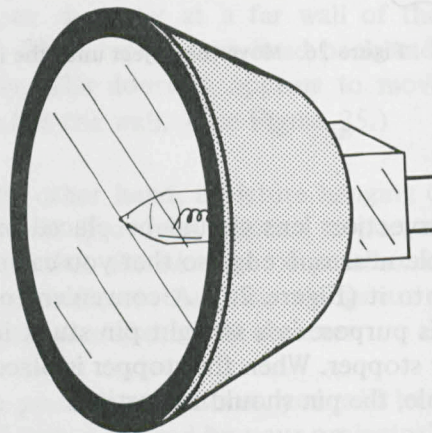


Figure 29. Projector lamp with image-forming mirror.

A mirror of this shape is called a *spherical mirror*, because it is shaped like a section of a large sphere. Its purpose is to project a real image of the lamp filament near the actual filament, to make the light source brighter and more uniform.

A spherical mirror acts much like a projection lens. It can form real or virtual images, depending on how large the object distance is compared to the focal length. Thus, you can repeat Experiment A-2, using a mirror instead of the lens. You will need a fairly large mirror, with a focal length of several inches, which can be used to project a picture of a slide on a screen.

Set up and proceed as in Experiment A-2. The only difference is that the image will be projected back past the object. You must place the screen beside or on top of the projector.

EXPERIMENT A-5 (OPTIONAL). Projecting 3-D Images with a Hologram

An ordinary slide provides a visual record with one major defect: it shows only a single view of the object. Though you may try to alter the view by looking at the slide from different angles or positions, you are limited to the view originally seen by the camera.

Holography

There is a method for making a visual record on a single, flat surface in which the viewpoint is flexible. This technique is called *holography*. The record, which is called a *hologram*, looks nothing like an ordinary slide, because there is no picture to be seen on it. Instead, there is a direct record of the original light in a special coded form, somewhat like the coded form of sound waves on a phonograph record. Attempting to see a picture by merely examining the hologram is like trying to hear music by examining the grooves of a phonograph record.

Wavefront Reconstruction

The process of using a hologram to recreate a visual image is called *wavefront reconstruction*. In effect, the light patterns which came from the original object are reproduced when light is passed through a hologram. Viewing these reconstructed patterns is much like

viewing the original object—the “picture” is even three-dimensional.

To understand wavefront reconstruction in detail requires knowledge of physical optics. On the other hand, viewing one is not difficult, especially with some newer types of holograms. What is required is to illuminate the hologram uniformly with a very bright light of a single color. A laser is best for this purpose, but you can do quite well by using your projector light and a colored filter. The astonishing result is well worth the effort.

Procedure

Set up the projector, with or without the lens, as shown in Figure 30. Mount the filter so that the projector light shines through it. An air gap is needed between filter and projector (about 1 in), so that the filter will not overheat.

Mount the hologram in the filtered beam a short distance away. Look through the hologram toward the projector until you locate the image. Try viewing it from the projector from various angles and positions. Test for parallax *with* the image by moving your head back and forth. See what happens when you remove the filter.

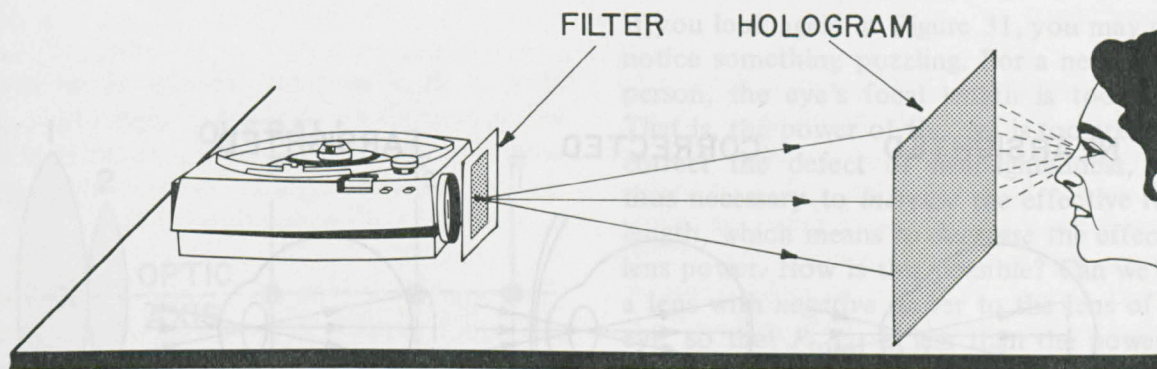


Figure 30. Using a projector to view a hologram.

DATA ANALYSIS AND DISCUSSION

By now you have learned a fair amount about the formation of images using lenses and perhaps mirrors. You have not yet learned *how* these devices do their job (that will be dealt with in the next section), but you have a knowledge of *what* they do. This knowledge can be applied to help you understand many situations of practical interest.

To apply what you have learned about lenses to a practical case, consider again how the human eye works.

Focal Length of the Eye

In Experiment A-2, you found that when a lens forms a real image of a distant object, the image distance approximately equals the focal length, f . More exactly, the focal length is always a bit less than the image distance.

From this result, what do you suppose is the focal length of your eye? The answer is easy: for your eyesight to be excellent, f must be slightly less than the diameter of your eyeball. Why? Because, for you to see clearly, the image must be in focus on the retina. Thus the image distance must equal the eye diameter. Incidentally, the diameter of an adult human eyeball is almost 2.5 cm (Figure 31).

Adjusting the Focal Length

How is it possible for an image to remain focused on the retina when the object moves closer to the eye? Some adjustment in the eye itself is needed.

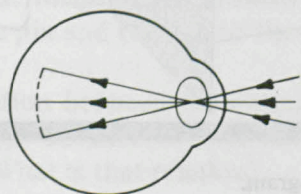
With a camera or a slide projector, this problem of focusing is solved by an adjustment of position of either the screen or the object (sometimes both). With the eye, of course, that is not possible. (Can you see why?)

Nature has solved the problem with typical ingenuity. The focal length of the eye changes as required. This process is called *accommodation*. To do it, eye muscles actually change the shape of the lens on command from the brain.

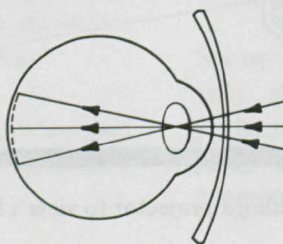
Correcting the Focal Length

When the eye's range of accommodation is not adequate—that is, when the focal-length changes are not sufficient—an additional, external lens may be needed. Together with the eye's lens, this external lens forms a lens combination with the correct focal length. The use of an external lens is called *correction*. It is the function of eyeglasses.

NEARSIGHTED



CORRECTED



FARSIGHTED

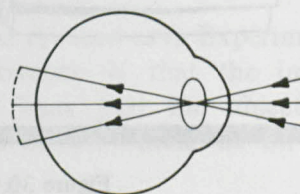


Figure 31. For perfect vision, the image is focused at the retina. (The eyes here are drawn roughly to life size.)

THE POWER OF LENSES

When your eyes are tested, and glasses are ordered to fit the particular need, a prescription is written for lenses of a certain *power*. The power P is the reciprocal of the focal length:

$$P = \frac{1}{f}$$

If f is measured in meters, then P comes out in *diopters* (D). Therefore, $1 D = 1 \text{ m}^{-1}$. This is the only common unit for lens power.

Notice that lens power is larger for smaller focal lengths and vice versa. A *high*-power microscope lens, for example, has a very *short* focal length. By contrast, ordinary window glass, which has “zero power” from an optics point of view, is in effect a lens of “infinite focal length.”

Powers Combine by Addition

What is the purpose of using P instead of f ? That is, why does the optician prefer to think “power” rather than “focal length”? After all, the two convey the same basic information.

The main reason is that, when individual lenses are put together to form combinations, *the lens powers add up* while the focal lengths do not. This is illustrated in Figure 32.

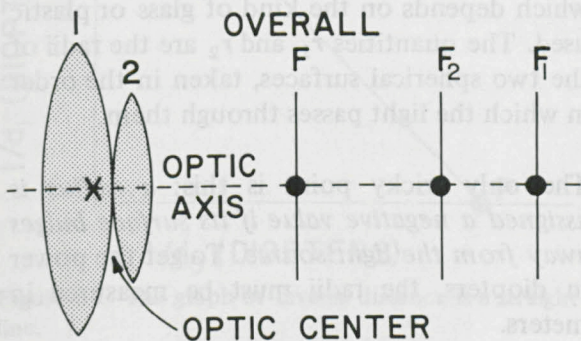


Figure 32. Total power $= P_1 + P_2$.

The figure shows two lenses placed side by side on the same optic axis. The two focal planes are also shown, marked F_1 and F_2 .

You can see that the overall focal length is not the sum of the individual lengths. On the contrary, it is actually *less* than either length alone. However, the total power is greater, the rule being:

$$P = P_1 + P_2$$

(To make sure that you understand this, you can check it by measuring distances on the figure, and doing the necessary calculations.) The rule applies whenever two or more *thin* lenses are placed very close together to form a single combination. A lens is said to be “thin” if its thickness is much less than the focal length and the image and object distances being measured.

Needless to say, the addition may be done in terms of the focal lengths, but then the expression for the overall focal length is a little more complicated:

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

This form shows why it is often more convenient to specify P than f for lenses.

The Power Can Be Negative

If you look again at Figure 31, you may now notice something puzzling. For a nearsighted person, the eye’s focal length is too short. That is, the power of the eye is too great. To correct the defect of nearsightedness, it is thus necessary to *increase* the effective focal length, which means to *decrease* the effective lens power. How is this possible? Can we add a lens with *negative* power to the lens of the eye, so that P_{total} is less than the power of the eye alone?

The answer is that we can. This is discussed on the next page.

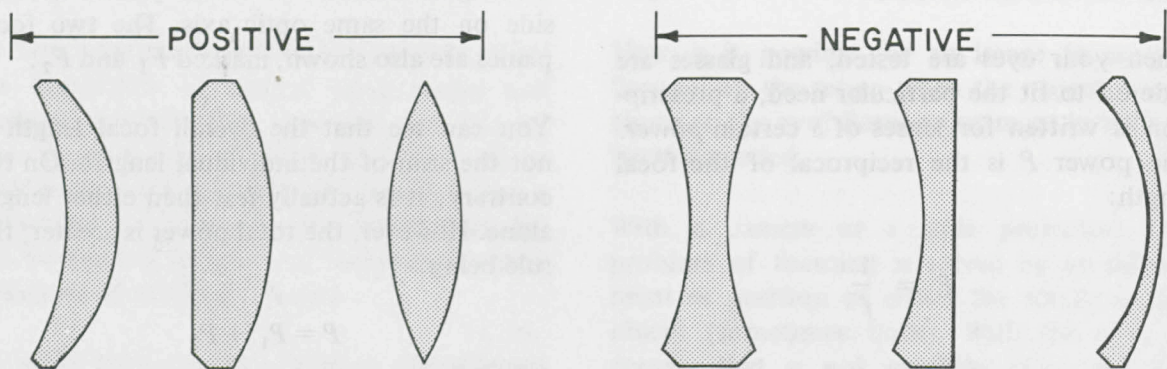


Figure 33. Lens power can be of either sign.

TYPES OF LENSES

Some lenses have the property that, when they are combined with other lenses, they always *increase* the total power of the combination. These are called *positive* lenses. There are several versions of this type, but they all are thicker in the center than at the edge (Figure 33).

The opposite is true of *negative* lenses. These are always thicker at the edges. They have the property that they *decrease* the power of lens combinations in which they are included.

Meniscus Lenses

You might think that you could decide whether a lens is positive or negative just by observing the curvature of its surfaces. Do they bulge inward (*concave* type), or outward (*convex* type)?

However, the lenses at the left and right ends of Figure 33 are “concavo-convex.” They bulge inward on one side and outward on the other side. These are often called *meniscus* lenses, from the Greek word *mene* for moon (because the shape may remind you of the crescent moon). They can be either positive or negative, and only the “thickness test” can tell which.

Meniscus lenses are always used in eyeglasses,

with the concave side toward the face. This permits free movement of the eyelashes, and yet allows the glasses to be quite close to the eyes. If you have a friend who wears glasses (or if you do), see if you can deduce the type of visual defect involved by examining the glasses themselves to see whether they are positive or negative. (Positive lenses correct for farsightedness.)

Lensmaker's Formula

The expert, grinding lenses to order, uses a very simple formula which gives the power of a lens in terms of the shape of its two surfaces.

$$\left(P = K \frac{1}{r_1} - \frac{1}{r_2} \right)$$

Here K is a constant (usually about 0.6) which depends on the kind of glass or plastic used. The quantities r_1 and r_2 are the radii of the two spherical surfaces, taken in the order in which the light passes through them.

The only tricky point is this: a *radius* is assigned a *negative* value if its surface bulges away from the light source. To get the power in diopters, the radii must be measured in meters.

THE LENS RULE

In Experiment A-2, using your projector's lens, you observed how the image position changed when the object distance was changed.

You now know that the *inverse* focal length, $1/f$ (the power P of the lens), is in some ways simpler to use than f itself. This suggests a question: what if inverse values are used for d_O and d_I as well? Does this also result in a simplification?

Plotting the Inverse Distances

Replot your data from Experiment A-2, this time using the inverse object distance, $1/d_O$ and inverse image distance, $1/d_I$. You will probably find the result surprising, and certainly much simpler than the original plot of d_O versus d_I . Except for experimental errors, the points for inverse distances all fall on a single straight line.

Draw the "best" straight line you can on this new plot. Use your judgment to determine the best line. You won't be able to hit all the points, but the best line will miss about as many above as below. Your graph should look much like Figure 34.

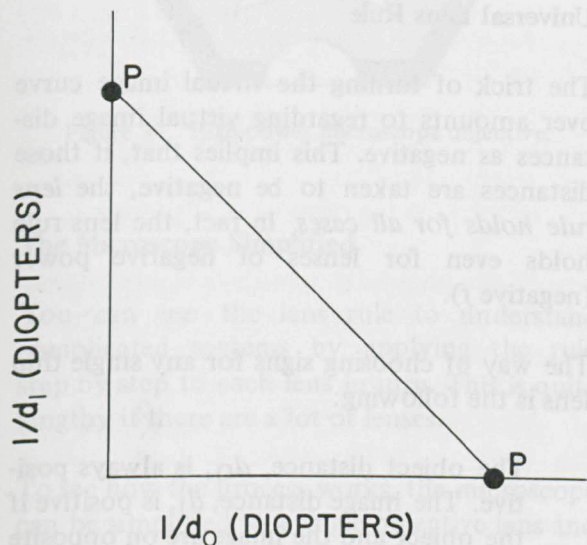


Figure 34. The graph of inverse distance is a straight line.

If the object is very far away, then $1/d_O \cong 0$ and the image falls upon the focal plane. That is, $d_I = f$ and $1/d_I = 1/f = P$. Thus, the place where the straight line crosses the vertical axis of the graph gives the power of the lens. What is the power of your lens?

Determine and record the power of your lens. Notice also that the straight line of the graph intercepts the horizontal axis at the same distance from the origin as it does the vertical axis. This means that, when $1/d_I = 0$, $d_O = f$. In other words, when the object is in the focal plane, the image is infinitely far away ($d_I = \infty$).

The Lens Equation

A careful analysis of Figure 34 gives the equation:

$$\frac{1}{d_O} + \frac{1}{d_I} = \frac{1}{f}$$

This is called the *lens rule* or the *thin lens equation*. It is the general relation between the object distance and image distance for *real images* formed by a *thin lens*. As before, the term "thin" means that the distance taken up by the lens along the optic axis must be small compared to the focal length f .

An Equivalent Equation (Optional)

Sometimes the lens rule is seen in another form, obtained by using object and image distances measured from the focal planes, instead of from the optic center. These distances are $x_O = d_O - f$ and $x_I = d_I - f$. Using these substitutions, the lens rule becomes

$$x_O x_I = f^2$$

This is the "Newtonian form" of the lens rule, named after Isaac Newton.

THE RULE EXTENDED

Using inverse distances to get the lens rule works so well for real images, why not try it again? In Experiment A-3, you were able to measure *virtual* image distances. Let us now try to find a lens rule for virtual images. The method for locating a virtual image (parallax method) was very different from that which worked for a real image (projection method). So perhaps you will expect the result to come out differently also.

Application to Virtual Images

Plot your data from Experiment A-3, using inverse distance $1/d_O$ versus $1/d_I$.

The data may not be as accurate as for the real images, and you don't have as many data points. Still, a trend should be evident. Figure 35 is an idealized plot for this case, similar to Figure 34 for real images.

Remarkably, the behavior is almost the same as for real images. To make comparison easier, the previous line for real images is shown dotted in Figure 35. The intercept is the same for both lines, at P , and each line makes the same angle (45°) with the horizontal axis.

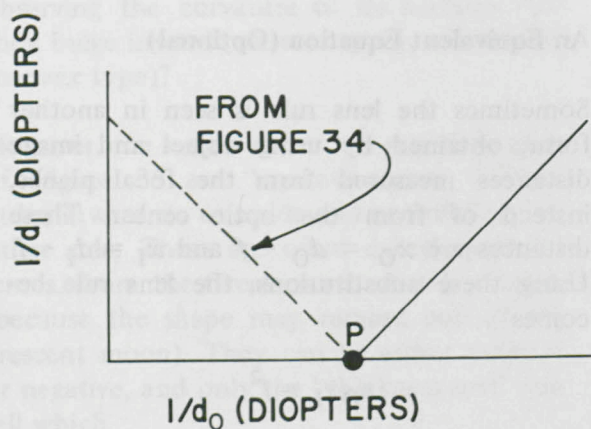


Figure 35. Plot the virtual image data on the same graph.

With one further adjustment, you can get a graph which is a single straight line. Just turn the new line over so that it points down rather than up. The result is shown in Figure 36.

If similar points were plotted on the same graph for a lens of a different power (focal length), the result would be a straight line parallel to the one shown, but displaced from it.

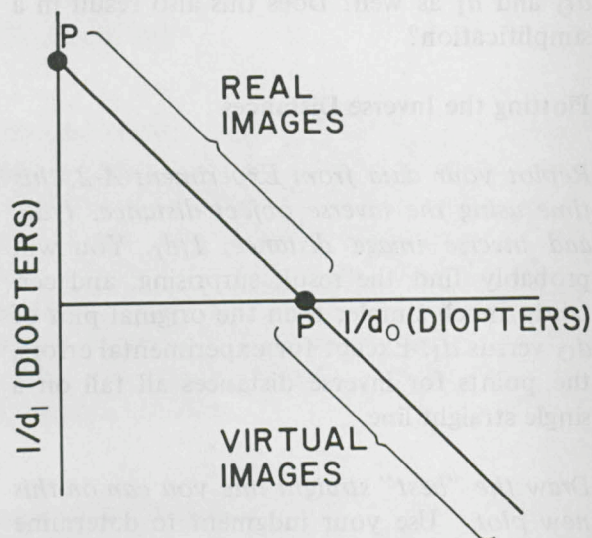


Figure 36. Reverse the virtual image graph to see the universality of the lens rule.

Universal Lens Rule

The trick of turning the virtual image curve over amounts to regarding virtual image distances as negative. This implies that, if those distances are taken to be negative, the *lens rule holds for all cases*. In fact, the lens rule holds even for lenses of negative power (negative f).

The way of choosing signs for any single thin lens is the following:

The object distance, d_O , is always positive. The image distance, d_I , is positive if the object and the image are on opposite sides of the lens, and negative if they are on the same side of the lens.

SYSTEMS OF LENSES

So far, all the discussion has applied only to a single thin lens or, at most, to a “thin” combination of lenses. However, in optical instruments there are often several lenses spaced apart by considerable distances to form a *lens system*. Moreover, the individual lenses may not be “thin” lenses.

The Microscope

Figure 37 shows part of a high-power microscope system. This is the *objective*, which is placed nearest the object to be viewed. The *eyepiece*, into which the viewer looks, is a separate system. In this case, the objective alone contains six individual lenses.

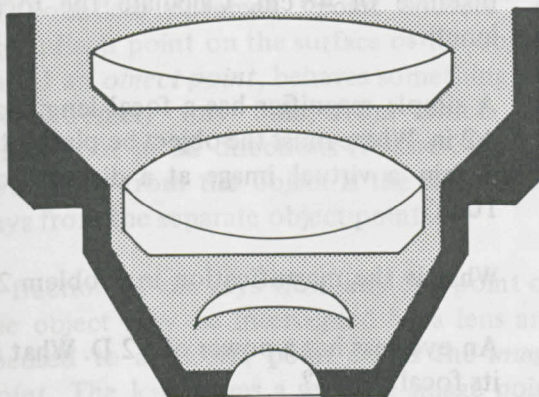


Figure 37. High-power microscope objective.

The Microscope Simplified

You can use the lens rule to understand complicated systems by applying the rule step by step to each lens in turn. This is quite lengthy if there are a lot of lenses.

To see how the process works, the microscope can be simplified to a single objective lens and a single eyepiece lens used as a magnifier (Figure 38). In fact, a basic microscope can be made in just this way.

The Microscope Analyzed

The optical chain now has two links, one for each of the separate lenses:

1. The object is located just *outside* the objective focal plane. A real image of the object is then formed by the objective.
2. The real image produced by the objective near the eyepiece serves as the “object” for the eyepiece. This “object” is just *inside* the eyepiece focal plane, and a final *virtual* image of the object is formed.

For each of these two steps, the “object distance” is almost equal to the focal length (f_1 or f_2). This makes d_I much larger than d_O , so the magnification d_I/d_O is large. The overall magnification is the *product* of the two separate magnification values:

$$M = M_1 M_2$$

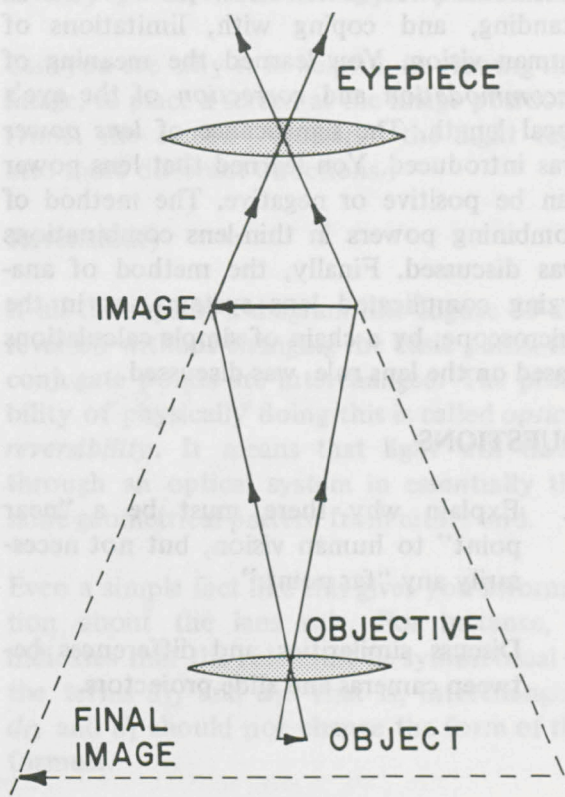


Figure 38. Principle of the microscope.

SUMMARY

In this section you explored some basic facts relating to the formation of images by slide projectors and similar optical devices. You learned the significance of the *focal length* of a thin lens, and how to measure that quantity. You also became acquainted with the concepts of *focal plane* and *optic center* of a lens or a lens combination.

You then learned about the differences between *real images* and *virtual images*. You learned methods for determining the image positions in each case. You found that the *lens rule* expresses the relation between object and image distances for both real and virtual images. In optional work, you may have investigated the relation between image formation by lenses and by mirrors. You found that the same ideas and methods can be used for each, with a few obvious changes.

You learned how these facts can be applied to situations of practical interest. You studied the usefulness of geometrical optics for understanding, and coping with, limitations of human vision. You learned the meaning of *accommodation* and *correction* of the eye's focal length. The significance of *lens power* was introduced. You learned that lens power can be positive or negative. The method of combining powers in thin-lens combinations was discussed. Finally, the method of analyzing complicated lens systems, as in the microscope, by a chain of simple calculations based on the lens rule, was discussed.

QUESTIONS

1. Explain why there must be a "near point" to human vision, but not necessarily any "far point."
2. Discuss similarities and differences between cameras and slide projectors.

3. If the sun suddenly emerges when you are taking snapshots on a cloudy day, should you increase or decrease the *f*-number setting? Explain.
4. What is the basic difference between a positive and a negative lens?
5. To get a larger projected image of a slide for a given "throw" to the screen, should you use a lens with longer or shorter focal length?
6. Can a negative lens ever produce a real image? (Hint: use the lens rule. Consider the sign of d_i .)

PROBLEMS

1. With an object located 16 cm from a thin lens, a real image is formed at a distance of 48 cm. Calculate the focal length.
2. A simple magnifier has a focal length of +2 in. Where must the object be placed to obtain a virtual image at a distance of 10 in?
3. What is the magnification in Problem 2?
4. An eyeglass has a power of -2 D. What is its focal length?
5. Calculate the power of a convex lens with surfaces of 30 cm radius. (Hint: use the lensmaker's formula.)
6. When photographing distant mountains, the film is located at 40 mm from the optic center of a camera lens, and the mountains are "in focus" on the film. What must the film-to-lens distance for sharp focus be when photographing a flower at a distance of 1.5 m?

SECTION B

Light Rays and Their Behavior

INTRODUCTION

A study of individual light rays forms the basis of geometrical optics. Even before studying ray behavior in detail, you can easily see the basic relation between rays and images. After a few experiments you should be able to answer further questions. For example, why must the lens rule have exactly the form that it does? Or, precisely what is the difference between a real image and a virtual image?

Image Points and Object Points

A basic step in relating images to rays is to think of an illuminated object as a collection of countless different “point sources” of light. Each point on the surface of the object, called an *object point*, behaves something like an extremely tiny bulb, from which rays stream out in all directions (Figure 39). The total light from the object is the sum of the rays from the separate object points.

A fraction of the rays from any one point on the object may be intercepted by a lens and focused to a second point called the *image point*. The lens forms a separate image point for each object point, and the two points together are called *conjugate* because they are joined by the light rays between them. The term conjugate means “joined.”

An image, in the ordinary sense, is composed of countless different image points, one for each point of the object. If the lens does its job well and without too much distortion, the image points are sorted out properly to produce a faithful copy of the original pattern.

By the way, you will notice by looking

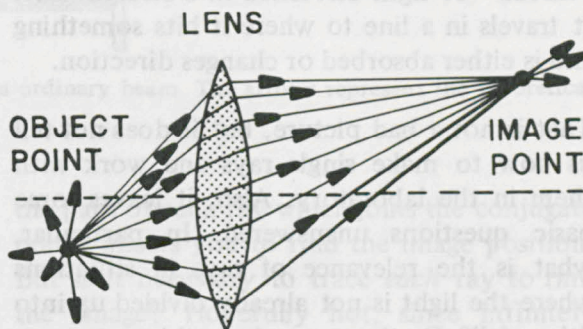


Figure 39. Conjugate points are joined by light rays.

carefully at Figure 39 that an image point is also a “source” of light. That is, rays go off in many directions from an image point, much as they go off from the object point itself.

Can you see why it is helpful, in viewing this image, to place a screen at the image position? (Hint: the screen “scatters” the light rays into more different directions.)

Reversibility

If all the rays in a diagram like Figure 39 are reversed without changing the basic paths, the conjugate points are interchanged. The possibility of physically doing this is called *optical reversibility*. It means that light will travel through an optical system in essentially the same geometrical pattern from either end.

Even a simple fact like this gives you information about the lens rule. For instance, it indicates that the rule must be symmetrical in the terms d_O and d_I . That is, interchanging d_O and d_I should not change the form of the formula.

RAYS IN THEORY AND PRACTICE

What is a ray? The word has been used several times throughout the module, probably without causing much trouble because most of us already have reasonable pictures of rays in our minds. A ray is usually imagined as a thin “thread” of light stretched in a straight line. It travels in a line to where it hits something and is either absorbed or changes direction.

That is not a bad picture, but it does not tell us how to make single rays and work with them in the laboratory. Also, it leaves some basic questions unanswered. In particular, what is the relevance of rays in situations where the light is not already divided up into “threads”? Furthermore, how thin can a ray be? Can it be as thin as you please, like an “ideal” mathematical line? Or is there some minimum achievable thickness in practice?

A Ray in Theory

For many purposes a ray can be regarded as a narrow beam, like a smaller version of the beam from a flashlight. But a more general meaning of “ray” is not limited to situations involving beams. Light travels through space like waves, and a ray can be thought of as an arrow which indicates the *direction of wave travel* at any place of interest. This meaning—

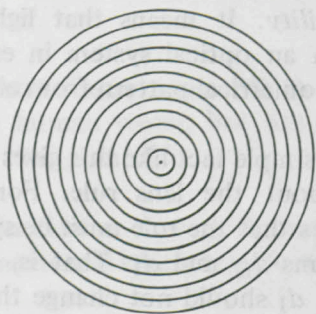
which applies to all types of waves—can be made clearer by thinking of water waves on a pond, since they are easy to visualize.

For example, Figure 40 shows a “point source” of water waves, as might be produced by dropping pebbles into a pond. At the left, the situation is represented in terms of concentric circles, indicating the various ripples in the wave. These expand outward in all directions as time passes. At the right, the *same* situation is represented by rays pointing outward from the center. This is to indicate that the basic motion is outward. The rays have meaning even though no beam exists in this case.

In the left picture, the ripples are indicated, but the motion is not, whereas in the right picture, the motion is indicated, but the ripples are not. Thus the two pictures complement each other. The ray picture has the advantage that it makes it unnecessary to try to visualize the nature of the waves, or even to remember that there are waves in most cases.

You should also realize that, because of the way waves travel, the rays are at right angles to the ripples. This is always true, no matter how complicated the wave motion may be.

WAVE PICTURE



RAY PICTURE

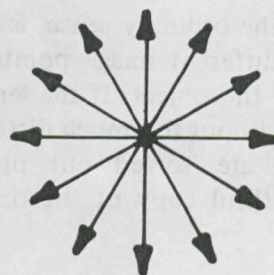


Figure 40. A point source of waves can be represented using two complementary pictures.

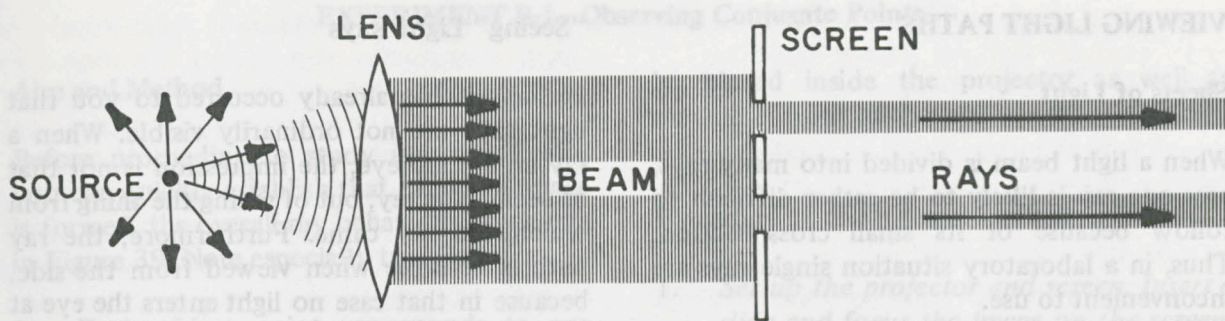


Figure 41. Rays can be made in the lab by division of an ordinary beam. The arrows represent the theoretical rays.

A Ray in the Laboratory

Turning now to the idea of rays as narrow beams of light, “practical” rays can be made in the lab by using the projector lens to first form a large beam. Then the large beam can be divided into smaller parts with a screen which has suitable holes (Figure 41).

Rays in this sense have certain limitations. For instance, they are dim because most of the light is thrown away. Furthermore, they have a minimum possible size because of diffraction (see Figure 6, page 5). When a beam goes through a tiny hole, the result may be *more* spreading than before, rather than less. This diffraction becomes extreme, and the beam is totally destroyed when the width of the hole is about the same as the spacing of individual ripples in the wave. For visible light, this is extremely small—less than 10^{-6} m.

It is essential to recognize that where practical rays do exist, as in Figure 41, they line up with the theoretical rays. Thus the lab rays tell us something about the theoretical rays, which is why they are useful.

Axial Rays and Chief Rays for Lenses

As mentioned before, making and studying rays in the lab can lead to a better understanding of image formation. Referring back to Figure 39, for example, the ability to trace

the path of each ray which joins the conjugate points allows you to find the image position. But is it necessary to trace *each* ray to find the image? Hopefully not, since infinitely many rays are involved, at least in theory! A little thought should convince you that tracing *any two rays* is sufficient. Two rays are enough because the image point, by its very nature, is where *all* the rays from the object point come together again.

Which rays should be chosen? Those which offer the greatest convenience. As shown in Figure 42, good choices are the *chief ray* and the *axial ray*. For thin lenses, the chief ray goes through the center of the lens, where the lens surfaces are nearly parallel. Thus the chief ray is not bent at all. An axial ray is one which is bent parallel to the optic axis. You will learn more about axial rays in your experiments.

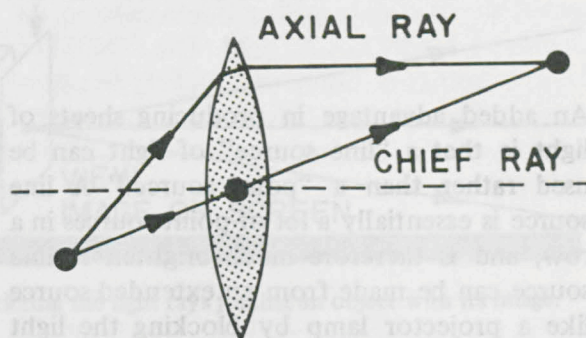


Figure 42. Tracing two rays is sufficient to locate an image.

VIEWING LIGHT PATHS

Sheets of Light

When a light beam is divided into many rays, any one ray is likely to be rather difficult to follow because of its small cross section. Thus, in a laboratory situation single rays are inconvenient to use.

For this reason and others, it is often preferable to work with "sheets" of light instead of rays. Each sheet is in effect a collection of many rays stacked on one another (Figure 43). Each sheet has a rectangular cross section, much longer than it is wide.

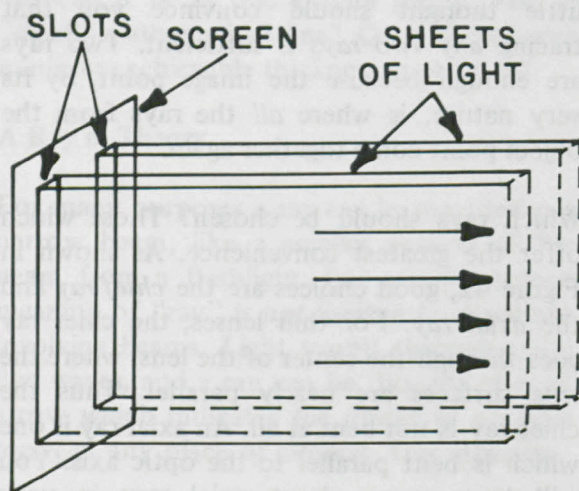


Figure 43. "Sheets" of light may be more convenient to use than rays.

An added advantage in producing sheets of light is that a "line source" of light can be used rather than a "point source." A line source is essentially a lot of point sources in a row, and is therefore much brighter. A line source can be made from an extended source like a projector lamp by blocking the light with a piece of cardboard with a narrow slit in it. This slit then acts as a line source and passes more total light than does a point-sized hole placed in the same position.

"Seeing" Light Rays

Perhaps it has already occurred to you that light rays are not ordinarily visible. When a ray enters the eye, the impression is not that of seeing the ray, but of seeing the thing from which the ray came. Furthermore, the ray path is invisible when viewed from the side, because in that case no light enters the eye at all.

You may have "seen" rays of light from a flashlight, a searchlight, a laser, or even sunlight. In each case, you actually saw light which was scattered *out* of the beam to your eyes by dust or smoke particles.

In your experiments you will use sheets of light instead of rays, but the same argument applies. One way to view them is to introduce smoke into the light paths, but smoke soon dissipates unless confined, and there are other inconveniences as well. On the other hand, a beautiful effect can be seen for a time. You might try pounding a chalkboard eraser in a light path to see the light scattered from the chalk dust.

A more convenient way to reveal the paths is by arranging to have the light skim along at a very slight angle to a flat piece of white paper (Figure 44). The line where the sheet intersects the paper becomes visible because the paper is being illuminated there. By moving the paper around, you can get a good idea of the overall geometry of the sheet.

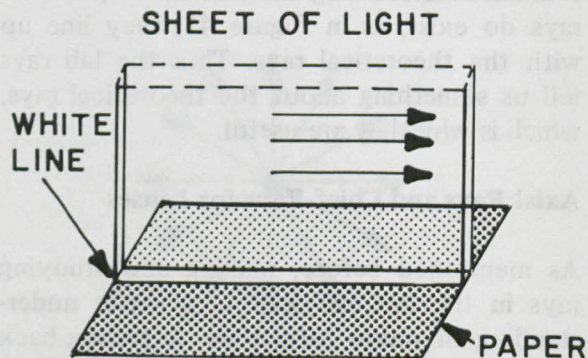


Figure 44. The sheet of light will make a bright line on a piece of white paper.

EXPERIMENT B-1. Observing Conjugate Points

Aim and Method

Before proceeding to study individual rays, you can quickly establish that, when an image is formed, the overall ray behavior is as shown in Figure 39. Note especially two features:

1. Each object point corresponds to one image point—no more, no less. This is rather obvious, but the projector makes it easy to “see.”
2. The whole lens—not just part of it—is used to form each different image point. This is far less obvious. Blocking off a section of the lens does *not* cut off a section of the complete image. Something different happens. What do you suppose it is?

The method of the experiment is suggested by Figure 45. The image of an ordinary slide is projected on a screen, as previously. Then a card with a small hole in it (“pinhole”) is used to select out some of the rays, while preventing all the others from passing through. The card should be about 2 in wide, so that it can

be placed inside the projector as well as outside in front of the lens.

Procedure

1. *Set up the projector and screen. Insert a slide and focus the image on the screen.*
2. *Insert the pinhole card, along with the slide, in the projector. Notice that all that remains of the original image is one bright spot. This can be moved around by moving the pinhole to various locations on the slide. If the pinhole is moved up, for example, does the spot move up or down? Does this agree with your notions of conjugate points?*
3. *Remove the pinhole card and place it in front of the lens. What happens to the image now? Is this reasonable in terms of your understanding of conjugate points? (Do you understand why placing the pinhole near the lens has the same effect as “stopping down” a camera by changing the f -number setting?)*

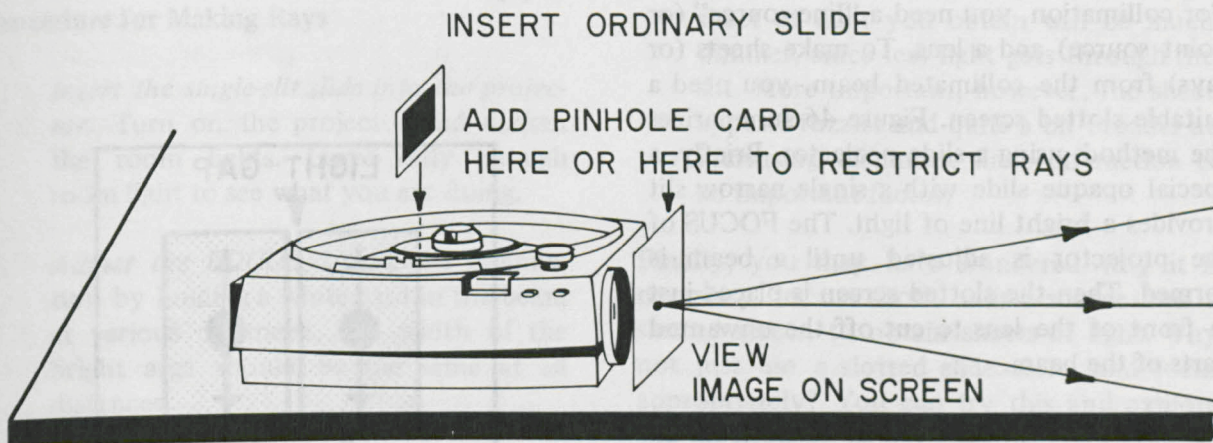


Figure 45. Conjugate points can be investigated by restricting the light rays joining an object with its image.

EXPERIMENT B-2. Forming and Viewing Light Rays

Aim of the Experiment

Forming light into a beam of parallel rays is called *collimation*. Causing an existing beam to converge to a point is called *focusing*. Your aim in this experiment will be to explore these fundamental processes of optics, especially collimation.

More generally, the purpose of the work is to give you some preliminary experience in manipulating light with the projector. The experiment should not take long (15-20 minutes), but you should make sure that you have a reasonable idea of what is happening before going on.

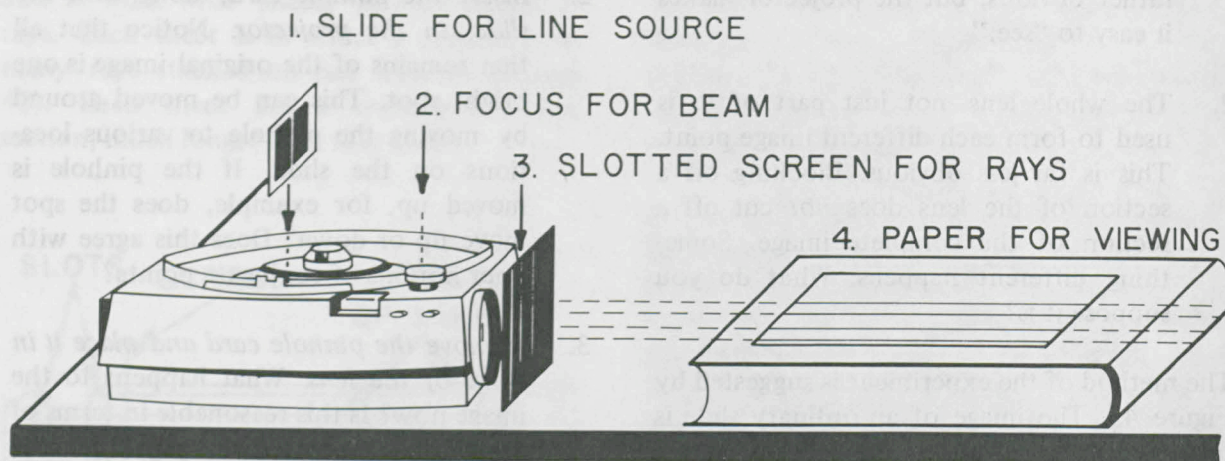


Figure 46. The projector can be used to manipulate rays.

Method

For collimation, you need a “line source” (or point source) and a lens. To make sheets (or rays) from the collimated beam, you need a suitable slotted screen. Figure 46 summarizes the method, using a slide projector. Briefly, a special opaque slide with a single narrow slit provides a bright line of light. The FOCUS of the projector is adjusted until a beam is formed. Then the slotted screen is placed just in front of the lens to cut off the unwanted parts of the beam.

The special “slit” slide can be made from an ordinary slide holder on which two razor blades are mounted in place of the usual transparency (Figure 47). There should be a slight gap between the blades—about the thickness of a sheet of paper—and the rest is

taped over so that light passes only through this gap.

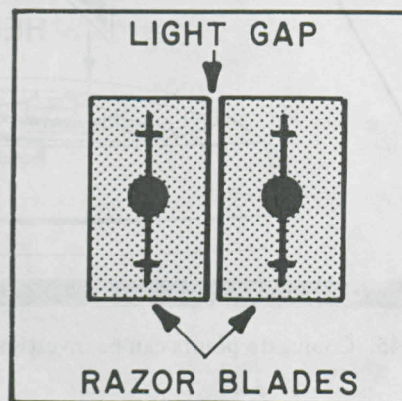


Figure 47. A special slide provides a “line source” of light. Tape covers the back side, except for the gap.

A simple and convenient slotted screen is provided by a 2-in-wide microscope slide, partly covered with strips of black tape (Figure 48). Each opaque strip is about 5 mm wide, and the clear slits between them are each 1 mm wide. This slotted screen can also be inserted in the projector, in place of the opaque slide, to see the effects and to compare them with those obtained using the much narrower single slit.

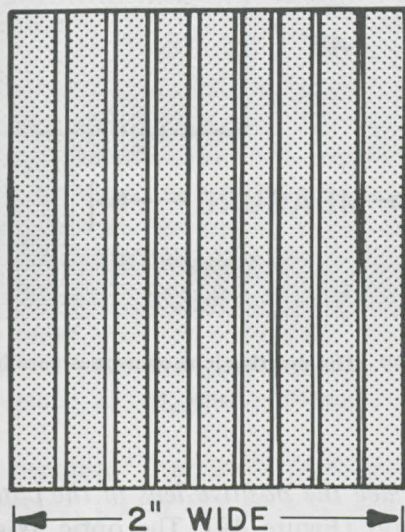


Figure 48. The screen for cutting a beam into sheets has one clear slot for each sheet.

Procedure for Making Rays

1. *Insert the single-slit slide into the projector.* Turn on the projector and darken the room lights. Leave only enough room light to see what you are doing.
2. *Adjust the FOCUS.* Check the collimation by holding a white card in the beam at various distances. The width of the bright area should be the same at all distances.
3. *Block the beam with the slotted screen.* Set up the screen very near the lens, as shown in Figure 46. The slots in the screen must be *parallel* to the slit in the slide.

4. *Observe the sheets of light.* Use a sheet of white paper for this purpose. Try moving the paper to sample the overall light geometry. Keep the paper as flat as possible to avoid unnecessary distortion. (It could be taped to a piece of cardboard.)
5. *Measure the "object distance" when the collimation is perfect.* Perfect collimation is indicated when the sheets of light are exactly parallel. As you may recall from earlier work, the object distance d_O is the distance from the slide to the optic center of the lens.

Reaching the Diffraction Limit

There should be very little spreading of each sheet of light in your collimated beam. However, if the slots in the slotted screen are too narrow—say only a few thousandths of an inch—then the spreading *increases* because of diffraction. You can easily observe this spreading.

6. *Replace the slotted screen with a second "slit" slide.* Compare the resulting light pattern with that in step 4. The single sheet of light you obtain will be much dimmer, since less light gets through the slit. More important, however, the sheet appears fuzzier and quite a bit broader at increasing distances, since diffraction is an important factor.

Finally, you may have wondered why it is necessary to use both a line source *and* a slotted screen to obtain sheets of light. Why not just use a slotted slide alone and focus appropriately? You can try this and explore the difficulties experimentally. Just remove the slide with a single slit and insert the slotted screen into the projector in its place. Try to get well-collimated sheets of light this way, observing them several feet from the lens. Can you see what the trouble is?

EXPERIMENT B-3. Bending Rays with Lenses and Mirrors

Aim of the Experiment

You now have some experience in making rays of light. (For simplicity, both rays and sheets will be called “rays” from here on.) At this point, you will try to understand how lenses and mirrors affect light rays in simple situations. Then you will be able to relate what you have observed to more complicated cases.

This experiment has two specific goals:

1. To observe and measure the *focusing of parallel rays* by a positive lens. This goal offers a clue to understanding the overall imaging properties of lenses.
2. To determine the *law of reflection*, which expresses the amount of bending of a light ray by a mirror surface.

Procedure for Goal 1

To accomplish the first goal, a positive lens with a known focal length is required. This lens should have a fairly large diameter, at least two or three inches, and the focal length should also be a few inches. An ordinary reading glass will serve nicely. The value of focal length f can be determined by the method of Experiment A-1.

Use the setup indicated in Figure 46, Experiment B-2. The book or other stand on which the viewing paper rests reveals the light paths at a small height above the table. This height should be comparable to the radius of the lens you will be investigating. If necessary, both the projector and the paper can be placed on stands to adjust the heights.

1. *Re-check the collimation, and adjust it if necessary.* Make sure that the rays visible on the paper are exactly parallel before they pass through the lens.

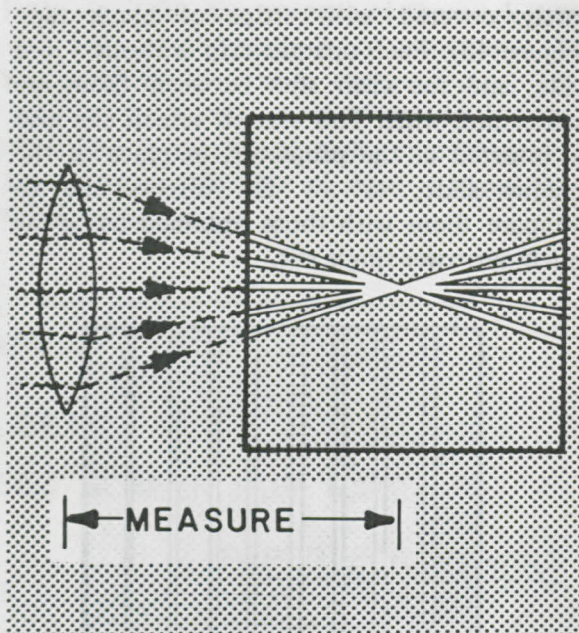


Figure 49. The positive lens converges parallel rays to a point.

2. *Place the positive lens in the path of the rays* (Figure 49). The optic axis should be horizontal and should point along the original ray paths.
3. *Adjust the lens height to reveal the convergence of rays on the paper.* When the appearance is similar to Figure 49, fix the lens in position. Use a ring stand and lens clamp, or another convenient method.
4. *Measure and record the distance of convergence of the rays.* This is the distance from the point of convergence to the lens plane. The distance from a point to a plane is taken *perpendicular* to the plane.

The data-taking for the first goal is now complete. However, the focal length f of the lens should be recorded for comparison. If this is unknown, it should be determined by

using the lens to form an image of a very distant object. Then the image distance, d_i , equals f . What is the relation between f and the distance of convergence for parallel rays?

Procedure for Goal 2

Again start with the setup of Figure 46. A piece of polar graph paper may now be used as the viewing paper to help measure angles. You also need a small flat mirror with at least one straight edge.

The mirror is placed upright on the viewing paper in the path of the rays. The rays strike the surface and are bent to a new direction. You will measure the relation between the *angle of incidence* (incoming angle) and the *angle of reflection* (outgoing angle). As shown in Figure 50, these are defined as angles which the rays make with the *normal direction*. The “normal” is just the perpendicular to the mirror surface, as indicated in Figure 51.

A “front-surface” mirror is preferable, but not necessary. Unlike the more familiar back-

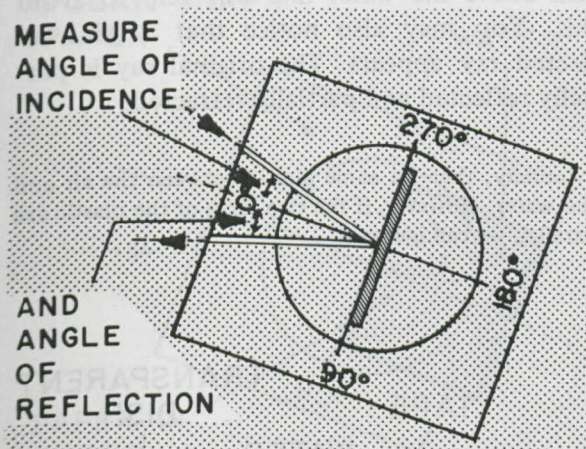


Figure 50. The mirror bends a ray from the angle of incidence to the angle of reflection.

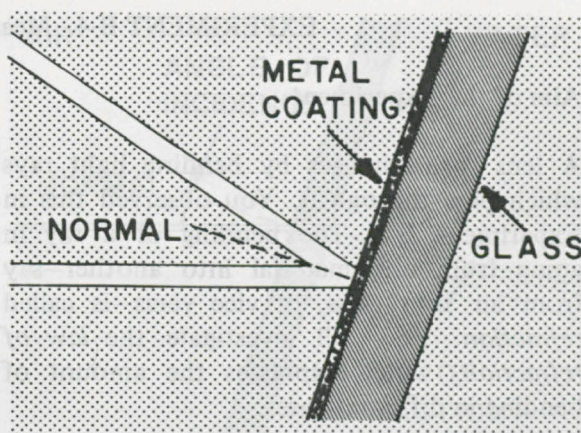


Figure 51. The “normal” direction is perpendicular to the mirror surface. (This shows a “front-surface” mirror.)

surface variety, a front-surface mirror reflects the light without making it travel through any glass (Figure 51). Thus there is less loss of brightness and less distortion.

1. *Re-check the collimation, then block off all of the rays but one.* Several rays at once here would merely cause confusion.
2. *Position the polar paper as suggested by Figure 50.* The incoming ray should skim along a radial line on the paper at a small angle (about 10°) to the line marked 0° .
3. *Place the flat mirror upright on the center of the paper.* The reflecting surface should lie along the line marked 90° and 270° . The line marked 0° then shows the normal.
4. *Measure the angle of incidence and angle of reflection.* These can be read directly from markings on the polar paper (Figure 50).
5. *Repeat steps 2-5 for several different angles of incidence between 0° and 90° .*

EXPERIMENT B-4. Measuring the Refraction of Light

Aim of the Experiment

A lens forms images by bending light rays which pass through it. You observed this in Experiment B-3. The bending of a ray in going from one material into another—say from air into glass, or vice versa—is called *refraction*. You will investigate the *law of refraction*, which expresses the amount of bending in different cases.

Method

The method will be to make a ray, using the projector, and to measure its refraction with a convenient setup. Using an ordinary lens for the refraction is not convenient for two reasons: the surface of the lens is curved, so the amount of refraction varies from one part to another, and a ray cannot be viewed while inside the glass, so its direction is not easy to determine there.

An alternative which avoids both difficulties is to use water instead of glass as the refractive material (Figure 52). The optical behaviors are similar, but the water can be easily contained in a flat-sided dish with viewing paper *inside* the dish to reveal the ray path. Ordinary paper will not do because it is not waterproof, but white plastic “contact paper” with an adhesive backing works very well. The measurement of angles in the water is easy if the contact paper does not cover the

whole bottom of the dish. Then polar graph paper under the transparent dish can be read by looking down through the water (Figure 53).

Procedure

The procedure here is very similar to that used for investigating the law of reflection in Experiment B-3. A ray goes from air into water, and both the “incoming” and “outgoing” angles are measured. Again, these are angles which the ray makes with the *normal direction*, defined as the perpendicular to the surface where the ray is bent (in this case, the air-water boundary* at the flat side of the dish). Here the outgoing angle, called the *angle of refraction*, is actually within the second material.

Again block out all but one of the rays from the screen to avoid confusion in the measurements. Another possible source of confusion is the appearance of a second ray in the water, due to light which comes in from above. A piece of opaque tape covering the side of the dish above the water line will help to avoid this. You may also notice that a faint reflected ray appears. The original ray is partially reflected and partially refracted.

*Actually, there is some plastic between the air and the water. However, if the two faces of the plastic are parallel, we can ignore it.

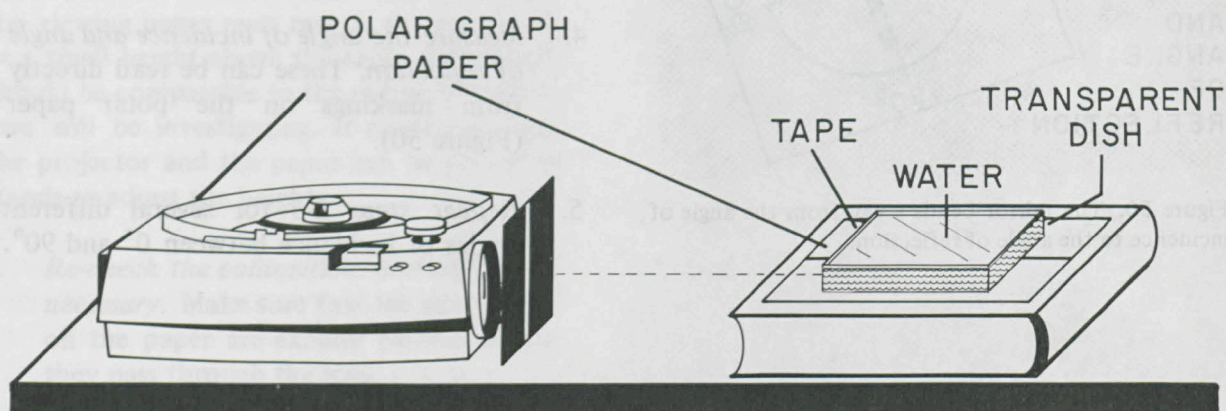


Figure 52. A small dish of water can be used for observing refraction of a ray.

1. *Recheck the collimation, then block off all rays but one.*
2. *Position the polar paper as suggested by Figure 53.*
3. *Place the dish containing water on the polar graph paper. One side should lie along the line marked 90° and 270° .*
4. *Measure the angles of incidence and refraction.* Note that the angle of refraction is measured from the line on the paper marked 180° . That is, the normal for the refracted ray points *into* the water (Figure 54).

If you find that you have trouble seeing the ray paths because they are too faint, adjust the heights of the projector and the dish. It helps to position the projector a little above the dish, pointing slightly downward. Sighting along the rays toward the projector also makes them appear much brighter. If neces-

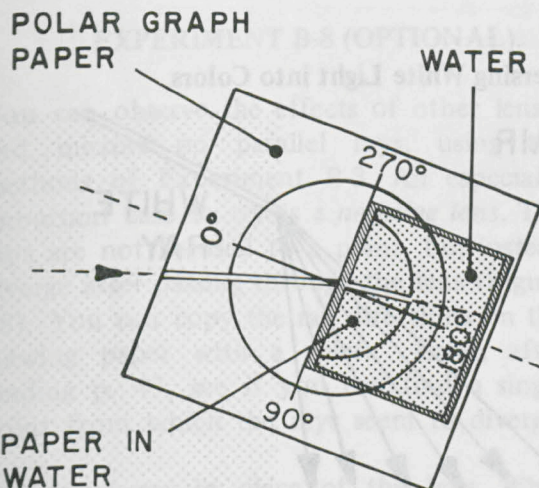


Figure 53. A ray is bent toward the normal in going from air into water.

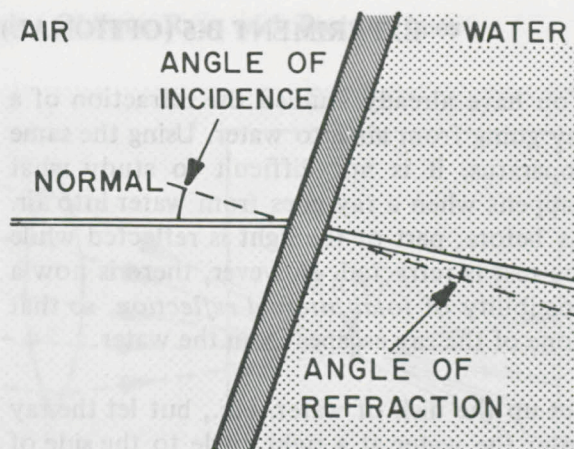


Figure 54. Angles are measured from the normal direction in both materials.

sary, you can replace the special slide with another having a slightly wider slit. This lets more light through the system, although the rays become less "sharp" as a result.

You should be able to see a faint trace of the ray after it goes out of the water again at the second side of the dish. Can you see that the ray is now *parallel* to its original direction? The bending at the second surface is exactly the reverse of that at the first surface (assuming the surfaces are parallel). Actually, this is just another instance of optical reversibility, as mentioned on page 29. Can you see why?

Other Refractive Materials

If extra time is available, you can try other liquids, such as mineral oil or glycerine, in place of the water. Or you can try the experiment with a piece of glass or plastic instead of a liquid. The edges must be polished so the ray can enter and leave effectively. The ray in the material can be seen if white contact paper is attached to the bottom face.

EXPERIMENT B-5 (OPTIONAL). Observing Total Internal Reflection

You have already studied the refraction of a ray going from air into water. Using the same apparatus, it is not difficult to study what happens when a ray goes from water into air. As before, part of the light is reflected while the rest is refracted. However, there is now a possibility of *total internal reflection*, so that none of the ray escapes from the water.

Set up the dish of water, etc., but let the ray enter the water at a right angle to the side of the dish (Figure 55). Place a small mirror in the water, so that the direction of the internal ray can be adjusted as desired. You can redirect the incoming ray to make a new angle of incidence inside the water. You will find that when this angle of incidence reaches a certain value, the angle of refraction (in the air) reaches 90° . This angle is the *critical angle of incidence*. For larger angles, the refracted ray vanishes entirely. How large is the critical angle for water, at least roughly?

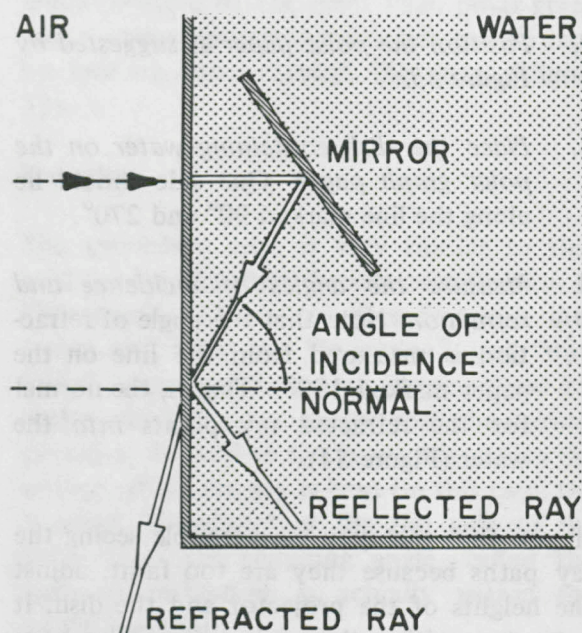


Figure 55. At the "critical angle of incidence," the refracted ray vanishes.

EXPERIMENT B-6 (OPTIONAL). Dispensing White Light into Colors

As you probably know, white light is composed of many different colors. These colors are bent by slightly different amounts when refracted. Thus, by refracting a white ray, you can separate the colors and view them individually (Figure 56). The process is called *dispersion*. Dispersion in water droplets causes rainbows.

You can see dispersion in the previous experiment by looking carefully at the refracted ray (Figure 55). This ray is no longer white, but it is spread out into a little "rainbow," especially when the angle of incidence is near 90° . Is the red light refracted more or less than the blue?

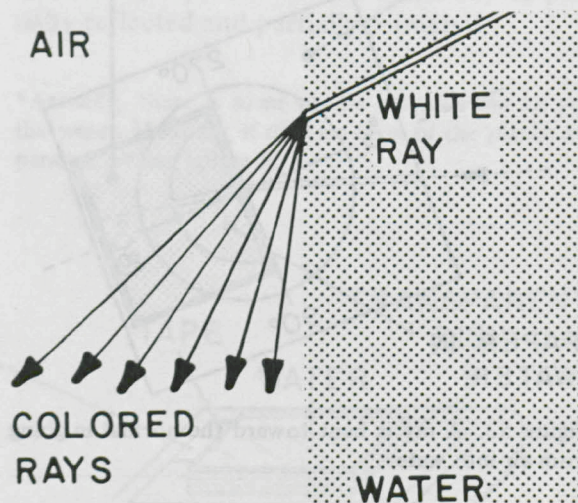


Figure 56. Different colors are refracted by different amounts. (The dispersion here is greatly exaggerated.)

EXPERIMENT B-7 (OPTIONAL). Focusing Oblique Rays with Positive Lens

In Experiment B-3, rays parallel to the axis, called *axial* rays, were focused to a single point by a lens. The point of convergence was on the optic axis. Rays which are *not* directed parallel to the axis of a lens are called *oblique*. Parallel but oblique rays also focus to a single point, but it is *off the optic axis* (Figure 57). To locate this point, you can repeat Experiment B-3, with the lens turned slightly so that the parallel rays are no longer axial. By trying a number of different angles, you can get a better idea of the overall focusing properties of the lens.

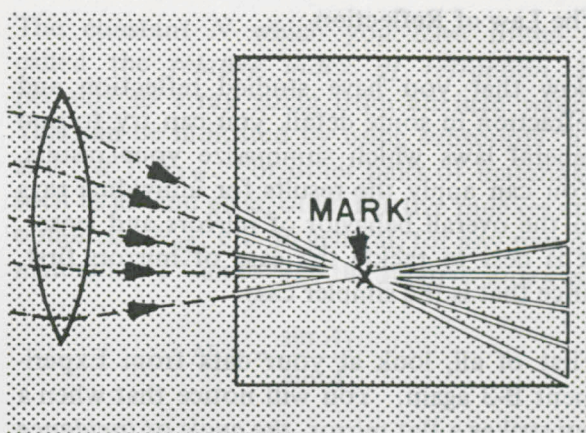


Figure 57. Oblique rays are focused off the optic axis.

Begin by having the parallel rays coming into the lens along the optic axis. Draw the optic axis on the paper and mark the point of convergence of the rays. Is this point on the optic axis? *Leaving the lens and paper in the same position*, move the projector so that the rays come to the lens from a slight angle. Mark the new point of convergence, and then

proceed to another angle. Do this for several angles on both sides of the optic axis. What happens when the angles get large (about 20 or 30°)?

EXPERIMENT B-8 (OPTIONAL). Bending Rays with Other Lenses and Mirrors

You can observe the effects of other lenses and mirrors on parallel rays, using the methods of Experiment B-3. An especially important case to try is a *negative lens*. The rays are not focused to a point, but instead diverge after passing through the lens (Figure 58). You can copy the ray directions on the viewing paper with a ruler. Later, after reading p. 47, see if you can find a single point from which the rays seem to diverge.

Try eyeglasses in place of the lens. What happens? Are the eyeglasses positive or negative? Also, try a concave mirror instead of the lens. The rays should converge after being reflected. If so, what is the distance of convergence?

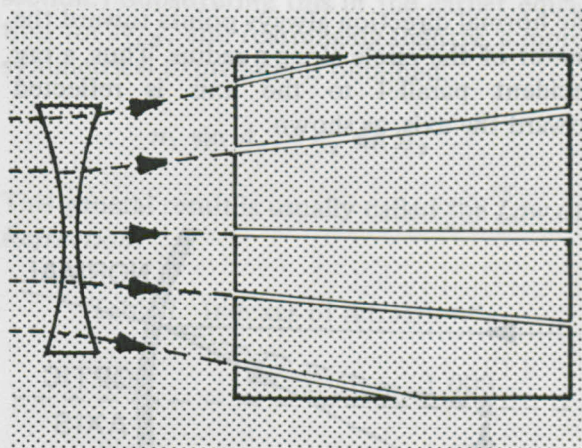


Figure 58. With a negative lens, the parallel rays diverge.

DATA ANALYSIS AND DISCUSSION

The Law of Reflection

Now that you have studied reflection and refraction in some detail, you can try to find general laws which summarize your results. You can apply the results for individual rays to the overall behavior of mirrors and lenses. It is best to tackle the case of mirrors first, since that is the simpler case.

First of all, look at your data from Experiment B-3 again. You will notice that when a ray strikes a flat mirror, *the angle of incidence equals the angle of reflection*. This simple relation is called the *Law of Reflection*. The angles are always measured from the normal direction.

Reflection from a Spherical Mirror

For *curved* mirrors, the normal changes direction from place to place. However, the same law applies as long as the normal is drawn to the place where a particular ray strikes. This is why rays which start out parallel may converge after reflection. For instance, one case of great practical interest is that of a spherical mirror. As Figure 59 indicates, the cross section of such a mirror is part of a circle, and the normal direction always points toward the center of the circle along a radius. Thus, a ray incident parallel to the axis is reflected toward the axis, as shown.

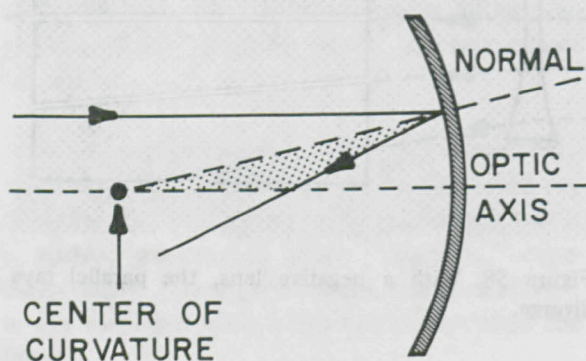


Figure 59. For a spherical mirror, every normal passes through the center of curvature.

Determining the Focal Length

For a spherical mirror, the geometry is relatively simple. As indicated in Figure 59, the incident and reflected rays make equal angles with the normal. Since the normal makes this same angle with the axis, the shaded triangle in the figure is isosceles (equal-sided). The two equal sides of the isosceles triangle are very nearly equal to one-half the distance between the mirror and the center of curvature. Thus the reflected ray crosses the optic axis at a point almost halfway between the mirror and the center of curvature. The same argument applies to other parallel rays which are not too far from the optic axis; they all cross the optic axis near the halfway point. Thus, *the focal length is one-half the radius of the mirror* (Figure 60).

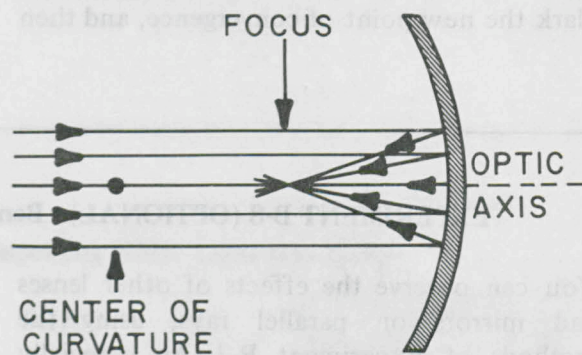


Figure 60. The focal point is determined by the law of reflection.

The farther the incident ray is from the optic axis, the greater the length of the equal sides of the isosceles triangle, compared to one-half the radius of curvature. Thus, rays farther from the axis are focused slightly closer to the mirror. Such an "error" in focusing is called an *aberration*. More will be mentioned about aberrations later.

ANALYSIS OF REFRACTION

The refraction of light is not quite so simple as reflection, as you can see by glancing again at your data for Experiment B-4. The angle of incidence is not equal to the angle of refraction. To see the relation between these angles, which will be called A_1 and A_2 , respectively, a plot of the data is helpful.

Plotting Your Data

It turns out that plotting A_1 versus A_2 very carefully all the way to $A_1 = 90^\circ$ leads to an unfamiliar curve. However, a plot of the *sines* of the angles is a straight line. This is illustrated in Figure 61, which shows also that the line goes through the origin. A graph of this type indicates that the two quantities are proportional, and the slope of the graph is the constant of proportionality. In equation form:

$$\sin A_1 = N \sin A_2$$

1. Plot your data as in Figure 61. Draw the best straight line you can through the points.
2. Find the slope N from your graph. The slope is the *rise*, the distance along the vertical axis between two points, divided by the *run*, the distance along the horizontal axis between the same two points.

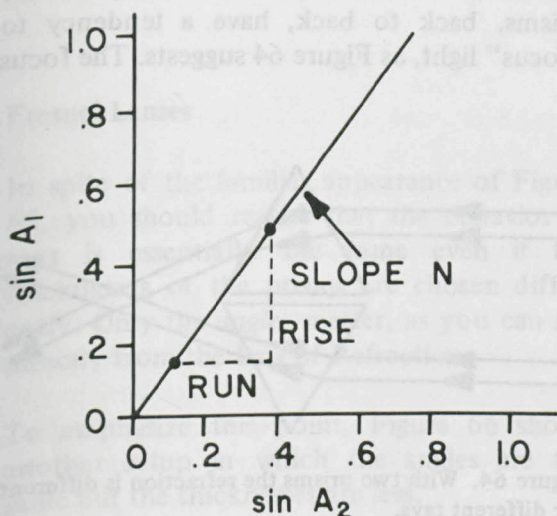


Figure 61. The sines of the angles of incidence and refraction are proportional

Index of Refraction

The constant N which you have found is called the *relative index of refraction*. It pertains to a ray going from air into water. This can be indicated by using subscripts "a" and "w": N_{aw} . For other pairs of materials, such as air and glass, other relative indexes are obtained: N_{ag} , etc. In general, for any pair of materials "1" and "2," the relative index is:

$$N_{12} = \frac{\sin A_1}{\sin A_2}$$

It is customary to specify the refractive properties of a given material, such as air, in terms of the relative index with vacuum. Vacuum is regarded as a standard "material" of incidence, and the subscript "v" is usually left off. A lower case letter is then used in place of N , and one speaks of the *absolute index of refraction* n . For air $n_a = N_{va}$, for glass $n_g = N_{vg}$, etc. Some typical values are given in Table I.

The Law of Refraction

From geometry, using the definitions of the various angles, one can show that, in general, $N_{12} = n_2/n_1$. (See Problems at end of section.) Substituting this in the earlier equation for N_{12} , and rearranging slightly, a more symmetrical expression is obtained.

$$n_1 \sin A_1 = n_2 \sin A_2$$

This general result is usually called the *Law of Refraction*.

Table I. Indexes of refraction*.

Air	1.003
Water	1.33
Plexiglas	1.49
Flint Glass (dense)	1.78

*For yellow light.

HOW LENSES WORK

Refraction in Window Glass

That lenses work by bending light rays should be clear from Experiment B-3. Even ordinary window glass bends a ray incident at an angle other than 0° . Yet a window is not a lens. What is the difference? The difference is that because the two surfaces of a window are parallel, a ray bent one way upon entering is bent back upon leaving (Figure 62). The ray ends up slightly displaced from its original line, but its *direction* is unchanged.

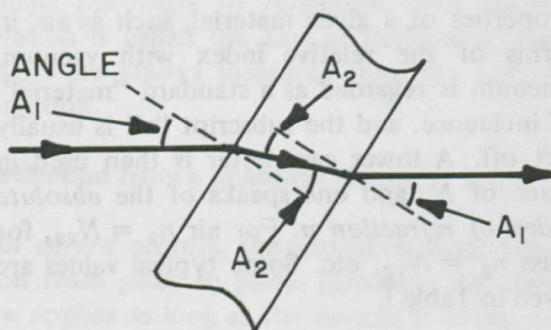


Figure 62. In window glass the two refractions cancel.

You can prove this by observing that the angle of refraction at the first surface (angle A_2) equals the angle of incidence at the second surface (also A_2) because the two normals are parallel. Then the “reversibility” of the Law of Refraction guarantees that the second refractive process (this time from material 2 to material 1) exactly restores the original angle A_1 .

Refraction in a Prism

What if the second surface is not parallel to the first? Then the ray does *not* exit in the original direction. This happens in “wavy” window glass, which explains the distorted view you get through such glass. It also happens in prisms and lenses, where the bending of rays is deliberate. To see this, look at Figure 63, which shows a ray going through a prism made of the same glass as the window in Figure 62.

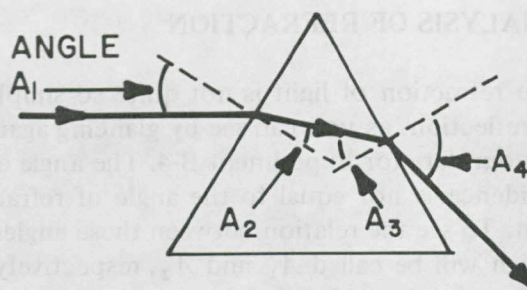


Figure 63. In a prism the two refractions do not cancel.

The angle of incidence is again A_1 , and, if the refractive indexes n_1 and n_2 are the same, the angle of refraction in the glass is again the same A_2 .

At the second surface, the situation is different. The normal is rotated, so the second angle of incidence is no longer A_2 , but some new angle A_3 . A second refraction occurs, with the overall result that the ray proceeds in an altered direction. This direction does not depend on the *thickness* of the prism, but only on the angle between the two faces. Thus the final angle, A_4 , is the same no matter where on a face the ray enters. However, because of dispersion, A_4 is different for the different colors of light.

Refraction in a Lens

A lens acts much like a number of different prisms connected together. Even just two prisms, back to back, have a tendency to “focus” light, as Figure 64 suggests. The focus

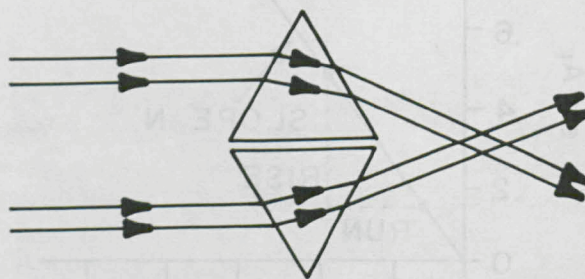


Figure 64. With two prisms the refraction is different for different rays.

is very imperfect, but it can be improved by increasing the number and variety of the prisms (Figure 65). The prisms *farthest* from the center should have the *largest* angle between their two faces, because the rays there should be bent most. At the center, the prism can have parallel sides, like a window glass, because the rays there should be bent not at all.

An ordinary spherical lens can be regarded as a sort of “limiting case,” in which infinitely many different prisms with various shapes have been put together to improve the focusing quality as much as possible. This is already clear from Figure 65, in which only five prisms, suitably chosen, look and act very much like an ordinary lens.

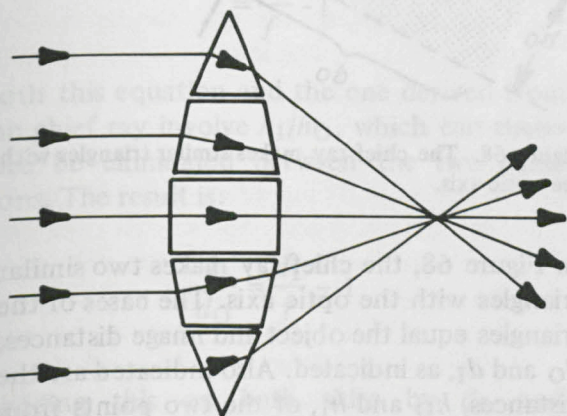


Figure 65. With many “prisms” the rays can be better focused.

Fresnel Lenses

In spite of the familiar appearance of Figure 65, you should realize that the behavior of rays is essentially the same even if the thicknesses of the prisms are chosen differently. Only the *angles* matter, as you can see directly from the Law of Refraction.

To emphasize this point, Figure 66 shows another setup in which the angles are the same but the thicknesses are less.

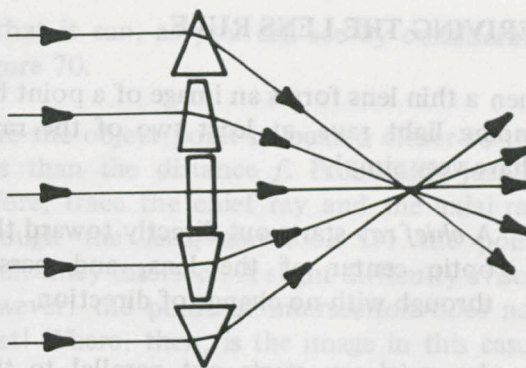


Figure 66. A “Fresnel lens” is like many thin prisms acting together.

The focusing ability is basically unchanged, even though the “lens” does not bulge out in the center as before. Real lenses of this type are rather common and are called *Fresnel lenses*. They are usually made from flat plastic sheets in which many circular grooves of the proper cross-sectional shapes have been formed. Lenses of extremely large diameter and short focal length are often made in this way.

Lens Aberrations

One of the main applications of optical theory is to determine the shape and composition of lenses which minimize errors of focusing. These errors are called aberrations. It is impossible, even in principle, to completely eliminate *all* aberrations by using a single lens.

Spherical aberration means that parallel rays farther from the optic axis are focused slightly closer to the lens. Spherical aberration is also a problem with spherical mirrors (see page 42). Another type of focusing error, which does not occur with mirrors, is *chromatic aberration*. The focal length of a lens is somewhat different for various colors because of slight color dependence of the index of refraction. Color-corrected, or *achromatic*, lenses are really two separate lenses glued together, one positive and the other negative. The two lens materials are chosen so that the chromatic aberration from one lens is almost exactly cancelled by the other.

DERIVING THE LENS RULE

When a thin lens forms an image of a point by bending light rays, at least two of the rays behave very simply:

1. A *chief ray* starts out directly toward the optic center of the lens, and passes through with no change of direction.
2. An *axial ray* starts out parallel to the optic axis, and is bent inward. It crosses the axis at a distance equal to the focal length f from the lens.

Ray Tracing (Graphical)

Both of the above facts are summarized in Figure 67. Because of optical reversibility, this diagram is equivalent to the diagram of Figure 42. Thus when a ray is “axial” in the earlier sense—namely it *ends up* parallel to the optic axis—it must have crossed the axis a distance f from the lens *before* entering the lens.

More importantly, because you know just where the chief ray and axial ray must go, such a diagram provides a graphical method for locating the image of a given object point. Locating images by following individual rays through an optical system is called *ray tracing*.

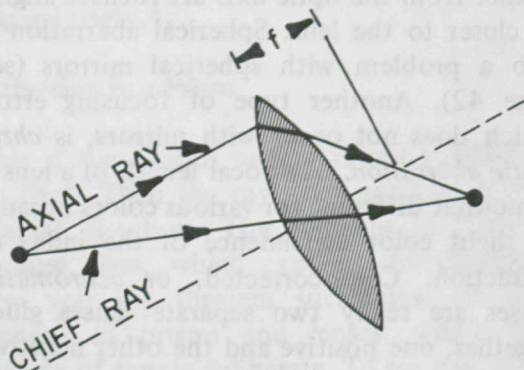


Figure 67. Ray tracing can be used to locate an image.

Ray Tracing (Analytical)

Besides being useful for graphical constructions, ray tracing is an easy method for calculating exactly where an image point must be. The method makes use of two pairs of similar triangles, as shown in Figures 68 and 69. In these figures the lens itself is not shown, but its position is indicated by a vertical line representing the lens plane. This is a reminder that, in the “thin lens” case, you can ignore complications due to the path length of the ray within the lens.

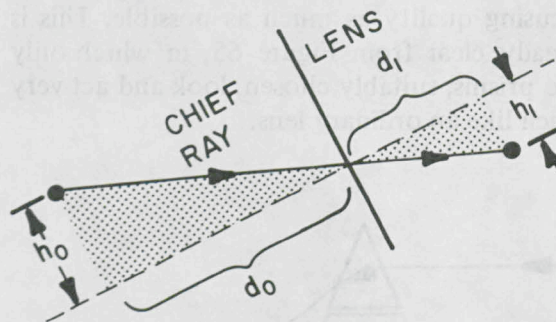


Figure 68. The chief ray makes similar triangles with the optic axis.

In Figure 68, the chief ray makes two similar triangles with the optic axis. The bases of the triangles equal the object and image distances, d_O and d_I , as indicated. Also indicated are the distances, h_O and h_I , of the two points from the optic axis. Because the triangles are similar, the ratios of corresponding sides are equal:

$$\frac{h_I}{h_O} = \frac{d_I}{d_O}$$

By the way, this shows that the *magnification*, h_I/h_O , has the value we assumed in Section A (Experiment A-2).

A second pair of similar triangles is formed by the axial ray and the optic axis. This is seen in Figure 69. Again the heights h_O and h_I are indicated, as well as the bases of the two triangles. The bases in this case are $d_I - f$ and f .

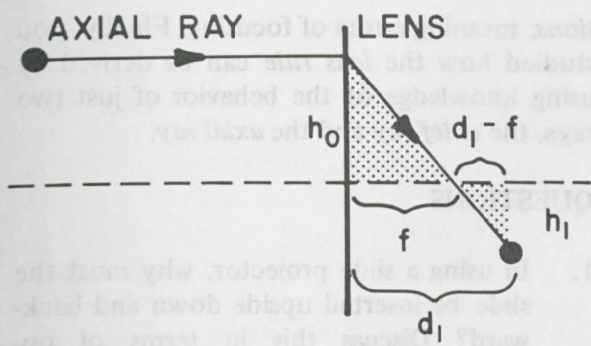


Figure 69. A second pair of similar triangles is made by the axial ray.

The ratios of corresponding sides are again equal, and

$$\frac{h_I}{h_O} = \frac{d_I - f}{f}$$

$$= \frac{d_I}{f} - 1$$

Both this equation and the one derived from the chief ray involve h_I/h_O , which can therefore be eliminated between the two equations. The result is:

$$\frac{d_I}{d_O} = \frac{d_I}{f} - 1$$

Dividing this on both sides by d_I , and rearranging the terms, you obtain the final result:

$$\frac{1}{d_O} + \frac{1}{d_I} = \frac{1}{f}$$

This will be recognized as the Universal Lens Rule found experimentally in Section A, proving (as advertised) that the lens rule can be derived using ray tracing methods.

Ray Tracing and Virtual Images

Besides giving the correct formula for the lens rule, ray tracing correctly indicates that the *real image* is *inverted*, and that the magnification has the predicted value. Can the method work as well for a *virtual image*? The answer

is that it can, as you can see by considering Figure 70.

Here the object point is located closer to the lens than the distance f . Proceeding just as before, trace the chief ray and the axial ray through the lens, and look for the point where they intersect. A slight difficulty arises, however: the point of intersection does not exist! Where, then, is the image in this case?

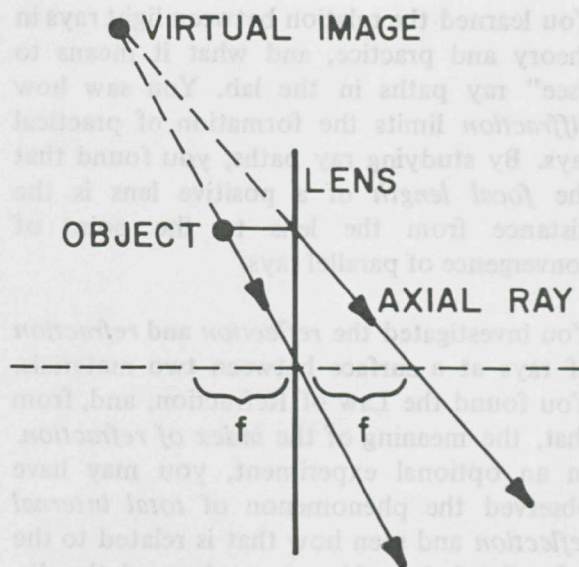


Figure 70. Ray tracing can be used for virtual images as well.

The image can be found by extending the two diverging rays backward until a point is reached from which they *appear* to be coming. If you imagine an eye, or other viewing device, in the path of the actual rays—to the right of the lens in Figure 70—you will see that the impression is just the same as if the rays really came from that point. The image is *upright*, as it should be, and on the same side of the lens as the object (“negative” image distance). However, if a screen were placed at the image position, it would show no focused image.

As an exercise, you can prove that, in this case as for a real image, similar triangles are made by the chief ray and axial ray with the optic axis. From these, you can derive the lens rule for virtual images.

SUMMARY

You have studied how image formation can be described using the behavior of individual rays. Complicated objects and images can be analyzed simply in terms of *conjugate points*. Furthermore, *optical reversibility* means that, in a sense, the conjugate points are equivalent; light rays starting from one conjugate point focus at the other.

You learned the relation between light rays in theory and practice, and what it means to “see” ray paths in the lab. You saw how *diffraction* limits the formation of practical rays. By studying ray paths, you found that the *focal length* of a positive lens is the distance from the lens to the point of convergence of parallel rays.

You investigated the *reflection* and *refraction* of rays at a surface between two materials. You found the Law of Refraction, and, from that, the meaning of the *index of refraction*. In an optional experiment, you may have observed the phenomenon of *total internal reflection* and seen how that is related to the refractive index. Also, you observed the *dispersion* of white light into colors by refraction.

From the discussion, you learned how curved mirrors and lenses focus rays because of changes in the *normal direction* of their surfaces. Some mention was made of *aberra-*

tions, meaning errors of focusing. Finally, you studied how the *lens rule* can be derived by using knowledge of the behavior of just two rays, the *chief ray* and the *axial ray*.

QUESTIONS

1. In using a slide projector, why must the slide be inserted upside down and backward? Discuss this in terms of ray behavior.
2. Can a *convex* spherical mirror ever form a real image? Show why, using the Law of Reflection.
3. A beam of parallel rays is focused to a point, using a lens. Describe this in terms of a ray picture.
4. Discuss differences and similarities between rays in the lab and rays in theory.

PROBLEMS

1. A ray goes from air into water through a flat plastic plate. Show that, for any angle of incidence in the air, the final angle of refraction in the water is the same as if the plastic were not present. (Hint: write down the Law of Refraction at each boundary.)
2. Using a ray diagram, locate the image of a point as seen in a plane mirror.

